INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

M.Sc. 4th SEMESTER (Applied Mathematics) EXAMINATIONS, 2014

Solid Mechanics III

(PGM - 403/1)

Full Marks: 70 **Time: 3 hrs**

(Symbols have their usual meanings)

Group-A

Answer question no. 1 and any Two from the rest in this group

1. **Answer any three :** **(3 x 5 = 15)**

a) Starting from the relations $f = \mu\varepsilon_{ij}\varepsilon_{ij} + \frac{\lambda}{2}\varepsilon_{kk}\varepsilon_{mm} - \gamma\,\varepsilon_{kk}(T - T_0) + G$ and $s = -\frac{\partial f}{\partial T}$, where f, s and T_0 are Helmholtz free energy function, entropy and absolute reference temperature respectively and G is a function of T, show that $s = \gamma\varepsilon_{kk} + C_\varepsilon \ln(\frac{T}{T_0})$, where $C_\varepsilon = T\frac{\partial s}{\partial T}$.

b) For a body occupying a region B, bounded by surface A with no body forces and free of external tractions, subjected to an arbitrary temperature distribution $\theta(x, y, z)$, show that the total change in volume is $\Delta V = \int_B 3\alpha_t\,\theta\,dv$, where α_t is the coefficient of linear thermal expansion.

c) In an uncoupled thermoelastic problem in isotropic media with time dependent source free temperature field, show that the Goodier's thermoelastic potential ϕ satisfies the equation $\left[\frac{\partial\phi}{\partial t} - \frac{\kappa\alpha_t(1+\sigma)}{1-\sigma}\theta\right]_{,ii} = 0$, where κ *and* σ are thermal diffusivity and Poisson ratio respectively.

d) When two Galerkin vectors are said to be equivalent ? If $\overline{F}_i$ *and* $\overline{\overline{F}}_i$ are two Galerkin vectors, show that $\overline{F}_i - \overline{\overline{F}}_i$ can be written in the form

$\overline{F}_i - \overline{\overline{F}}_i = (1-2\nu) f_{i,jj} + f_{j,ji}$. It is given that the displacement field u_i can be expressed in terms of Galerkin vector (F_1, F_2, F_3) in the form $2\mu u_i = 2(1-\sigma)\nabla^2 F_i - F_{j,ji}$, where in absence of body force, F_i satisfy biharmonic equation.

2. In a homogeneous isotropic medium, express the displacement components in terms of Galerkin vector (F_1, F_2, F_3) and show that the Galerkin vector satisfies the equation $\nabla^4 F_i = 0$, in absence of body force. Also determine τ_{kk} in terms of Galerkin vector. **(15)**

3. In the torsion problem of a cylindrical bar of length l, whose axis coincides with the axis of z, (z = 0 is fixed and z = l is twisted by a couple), show that the potential energy can be expressed in the form $V = \frac{1}{2}\mu\alpha^2 l \iint_R \left[\left(\frac{\partial \overline{\psi}}{\partial x}\right)^2 + \left(\frac{\partial \overline{\psi}}{\partial y}\right)^2 - 4\overline{\psi}\right] dxdy$, where $\overline{\psi}$ is the stress function. Discuss how will you determine the minimum potential energy and stresses using Rayleigh-Ritz method ? It is given that $\tau_{zx} = \mu\alpha \frac{\partial \overline{\psi}}{\partial y}$, $\tau_{zy} = -\mu\alpha \frac{\partial \overline{\psi}}{\partial x}$ and rest stress tensors are zero and also the medium of the cylinder is considered to be homogeneous isotropic . **(15)**

4. a) It is given that Airy's stress function F can be expressed as $F = \text{Re}[\bar{z}\phi_1(z) + \chi(z)]$, where $\nabla^2 F = P = 4\frac{\partial p}{\partial x}$ and $\phi_1(z) = p(x,y) + iq(x,y)$, $\chi(z) = p_1(x,y) + iq_1(x,y)$ are analytic functions of z where $p_1 = F - px - qy$. Show that in a state of plane strain the displacement components of a thermoelastic problem can be expressed as

$$2\mu(u+iv) = (3-4\sigma)\phi_1(z) - z\overline{\phi}_1'(z) - \overline{\chi}'(z) + 2\mu\left(\frac{\partial \phi}{\partial x} + i\frac{\partial \phi}{\partial y}\right),$$ where $\phi(x,y)$ is the Goodier's thermoelastic potential.

[It is assumed that the material is isotropic and bar represents the complex conjugate.]

b) Express strain tensors in terms of stress tensors and temperature in homogeneous isotropic medium. **(11+4)**

Second Half

Use separate answerscript

One mark is reserved for general proficiency

Answer any two (2) :- [12×2]

1. Derive the Lagrange's equation for bending of a thin plate.
2. Find the expression of bending moments for the bending of a circular plate when the boundary is clamped .
3. A simply supported rectangular plate is under the action of a load which is distributed according to the formula $p(x,y) = \frac{p_0 x}{a}$. Find the deflection and moments generated in the plate.