M.Sc. (Applied Mathematics) 4th Semester Examination: 2014

Classical Mechanics and Classical Electrodynamics

(PGM - 401)

Time- 3 hours **Full Marks-70**

Answer any six questions taking three from each group

Two marks are reserved for general proficiency

Use separate answer script for each group

The symbols used have their usual meaning

GROUP - A

1. a) Define Generalized Co-ordinates and degrees of freedom of a dynamical system. Derive Lagrange's equation of motion for simple dynamical system.

   b) A uniform rod of mass 3m and length 2l, has its middle point fixed and a mass m attached at one extremity. The rod when in horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{2ng/l}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}[\sqrt{n^2+1}-n]$ and will then rise again.

   [(1+1+5)+4]

2. a) Using Lagrange's equation of motion show that the angular momentum is conserved when the rotation coordinate q_k is cyclic .

   b) Derive Hamilton's equations of motion for a simple pendulum.

   [6+5]

3. a) For a simple dynamical system A = $\int_{t_0}^{t_1} T\, dt$ has a stationary value in the actual path as compared with some neighbouring paths, provided H, the Hamiltonian is constant throughout the actual path.

   b) A particle of unit mass moves along the x axis under a constant force f starting from rest at the origin at time t= 0. If T and V are Kinetic and Potential energies of the particle, respectively: Calculate $\int_0^{t_0}(T - V)dt$ and evaluate for the varied path in which the position of the particle is given by $x = 0.5ft^2 + \varepsilon ft\,(t-t_0)$, where ε is a constant. Show that the result is in agreement with Hamilton's Principle.

[5+6]

4. a) State and prove Liouville's theorem.

   b) For what values of α and β do the equations $Q = q^{\alpha}\cos\beta p$ and $P = q^{\alpha}\sin\beta p$ represent a canonical transformation? Hence find the generating function.

[(1+5)+5]

5. a) Show that relativistic form of Newton's second law is
$$F = m_0\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\frac{dv}{dt}.$$
Hence find the relativistic expression for the kinetic energy of a particle moving with velocity v and whose rest mass is m_0. Finally show that the total energy E of a relativistic particle is $E^2 = p^2c^2 + m_0^2c^4$.

   b) Define proper time interval between two events. How does it change under a Lorentz transformation? Hence derive an expression for time dilation.

[(2+3+1)+(1+2+2)]

Group – B (PGM-401)

6. (a) What is the 'Flux Rule'? Discuss it in the context of motional emf.

   (b) Find the flow of current in a circuit with an inductance, a resistance and a source of constant emf.

7. (a) State the Maxwell's equations. Show that the conservation of charge follows from these equations.

   (b) Show that the following equations

$$\square^2 A = -\mu_0 J \text{ and } \square^2 V = -\frac{\rho}{\epsilon 0}$$

   hold with appropriate choice of gauge.

8. (a) What is canonical momentum $\vec{p}$ of a particle in an electromagnetic field? Show that $\frac{d\vec{p}}{dt}$ is obtainable from a velocity dependent potential.

   (b) Find the expression of mutual inductance for two loops carrying constant currents I_1 and I_2 respectively.

9. (a) Find the expression of the electric potential of a charge distribution at a large distance from it in a series expansion.

   (b) Find the expression of energy due to a discrete finite charge distribution.

10. (a) Discuss the cyclotron motion of a massive charged particle placed in a uniform magnetic field.

    (b) Find the expression of the magnetic vector potential due to a small current loop at a very large distance from it.