BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

M. E. 1st Semester Final Examination, 2013

Discrete Structures
(M-902)

Full Marks 70 **Branch- ETC**

Time: 3 hours

Use separate answerscripts for each group.

GROUP A

Answer **any two questions** in this group.

Credit will be given for answers which are brief and to the point. Marks might be deducted for longwinded answers and irrelevant details.

1. (a) State and prove Lagrange's theorem.

   (b) Define homomorphism and kernel of a homomorphism. Show that the kernel of a homomorphism is a subgroup of the group on which the homomorphism is defined. Is it a normal subgroup? Justify you answer.

   (8+6)

2. (a) Let H be a normal subgroup of a group G. Denote the set of all cosets $\{ aH \mid a \in G \}$ by G/H and define $*$ on G/H by $(aH)*(bH) = ab\,H$ for all $aH, bH \in G/H$. Then prove that $(G/H, *)$ is a group.

   (b) Give an example to of an infinite group G having an infinite normal subgroup H such that the quotient group G/H is finite.

   (8+6)

3. (a) Prove that any field is an integral domain.

   (b) Is the converse true ? Justify your answer with a suitable example.

(c) Can you suggest an additional condition to be imposed on the integral domain so that the converse holds? Present your proposal in the form of a theorem. Provide a rigorous proof of this theorem.

(3+3+8)

Group B

Answer any **THREE** questions

4. a) Define poset. Show that the set of all rational numbers with usual order '$\leq$' is a poset. Is it totally ordered ?

   b) Let N be the set of all positive integers, $\wedge$ and $\vee$ are defined as $a \wedge b$=HCF of a and b, $a \vee b$=LCM of a and b. Prove that $(N, \wedge, \vee)$ is a lattice.

   c) Define a distributive lattice. Show that in a distributive lattice $(L, \wedge, \vee)$, $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$.

4+5+5

5. a) Define a Boolean algebra. Prove that $(P(S), \cup, \cap)$ is a Boolean algebra.

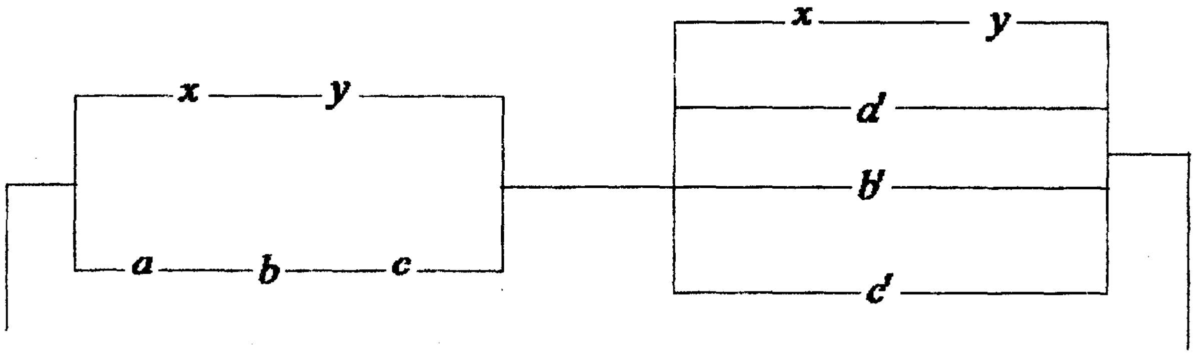
   b) Define a Boolean function. Find the conjugative normal form for the function $f = xyz + x'yz + xy'z' + x'yz'$.

   c) Using truth table show that

$$xy' + xy + x'y = x + y.$$

5+5+4

6. a) Simplify the circuit



b) A committee consists of a chairman, president, secretary & treasurer. A motion passes if and only if it receives majority of votes or the vote of the chairman plus one other member. Each member presses a switch to indicate its approval of motion. Design a switching circuit in such a way that the current passes if and only if the motion is approved.

c) A man observes the following rules during his meal.

i) If he takes coffee he does not drink milk.
ii) He takes biscuits only if he drinks milk.
iii) He does not take eggs unless he eats biscuits.

If it is known that he had coffee at certain meal can you say whether he had eggs in that meal.

4+5+5

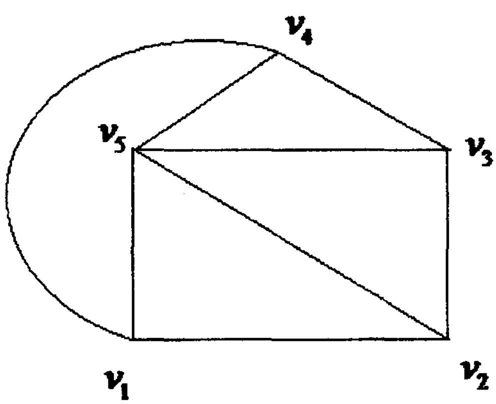
7. a)Define a binary tree. Prove that the number of pendant vertices in a binary tree is $\frac{n+1}{2}$, where n is the number of vertices in the tree.

b) Define a spanning tree. Show that every connected graph has at least one spanning tree.

c) Prove that the necessary condition for a simple connected graph to be planar is $e \leq 3n-6$.

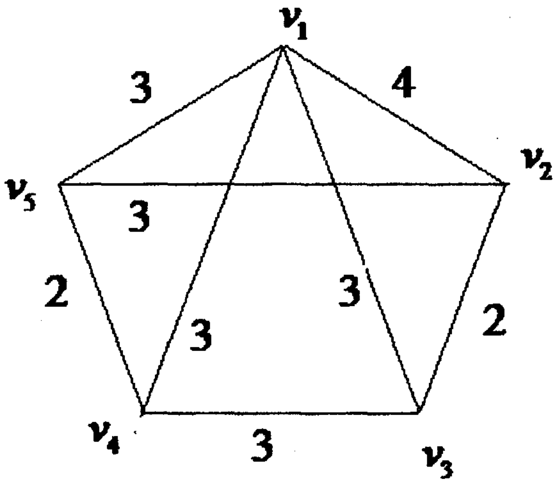
5+4+5

8. a) Find the spanning tree of the graph



Also determine the number of fundamental circuits.

b) By Prim's algorithm find a minimal (shortest) spanning tree in the following graph and also find the corresponding minimum weight.



6+8