BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

M.E. 1st Semester Final Examination, 2012
Discrete Structures (M-902)

Full Marks- 70 **Branch-ETC**

Time – 3 hours

All questions carry equal marks.

Group A

Answer any **TWO** questions from this group.

1.a) Define a field and an integral domain. Prove that a field is an integral domain but not conversely.

b) State the pigeonhole principle and utilize it to prove that a finite integral domain is a field.

7+7

2.a) State and prove Lagrange's theorem in the context of groups.

b) Give an example to show that the index of H in G may not always be computed using the formula $\circ(G)/\circ(H)$. (Here G is a group and H is a subgroup of G.)

7+7

3.a) Given any positive integer n, show that there exists an abelian group of order n.

b) Consider the set S= $\mathbb{Q}$-{1}, where $\mathbb{Q}$ is the set of rational numbers. Define, on this set, a binary operation * as follows:

$a * b = a + b - ab$.

Prove that (S, *) is an abelian group. Explain why it is necessary to delete the element 1 from $\mathbb{Q}$ in ordere to preserve the group structure.

7+7

Group B

Answer any **THREE** questions.

4 (a) Define partially ordered set. Illustrate with an example.

(b) Let S be a set and $P(S)$ be its power set. Prove that $P(S)$ is a lattice with respect to the operations $\cap$ (intersection) and $\cup$ (union).

(c) Prove that intersection of two sub lattices is a sub lattice. Give an example to show that union of two sub lattices may not be a sub lattice.

4+5+5

5 (a) Prove that a Boolean algebra (B, +, •) is a lattice with respect to the operations '+' and '•'.

(b) Prove that for every element 'a' in a Boolean algebra (B, +, •), $a + a = a$ and $a \bullet a = a$.

(b) Express the following Boolean functions in its disjunctive normal form:

(i) $(x + y)(x + y')(x' + y)$,

(ii) $(xy' + xz)' + x'$.

4+4+(3+3)

6 (a) Draw the switching circuit represented by the Boolean function

$xyz' + x'(y + z')$.

(b) A light in a room is to be controlled by three switches located at three entrances. Design a simple series-parallel switching circuit such that flicking any one of the switches will change the state of the light.

(c) A guest come to a hotel and said I am very particular about my food. Please obey the following rules when serving meal:

(i) At any meal when you do not serve bread you must serve ice-cream.

(ii) If you serve both bread and ice-cream, you must not served pickles in that meal.

(iii) If pickles are served or breads are not served, then ice-cream must not be served.

The manager tries to simplify these rules with Boolean algebra. What rule did he get?

2+6+6

7 (a) Define walks, paths and circuits in a graph.

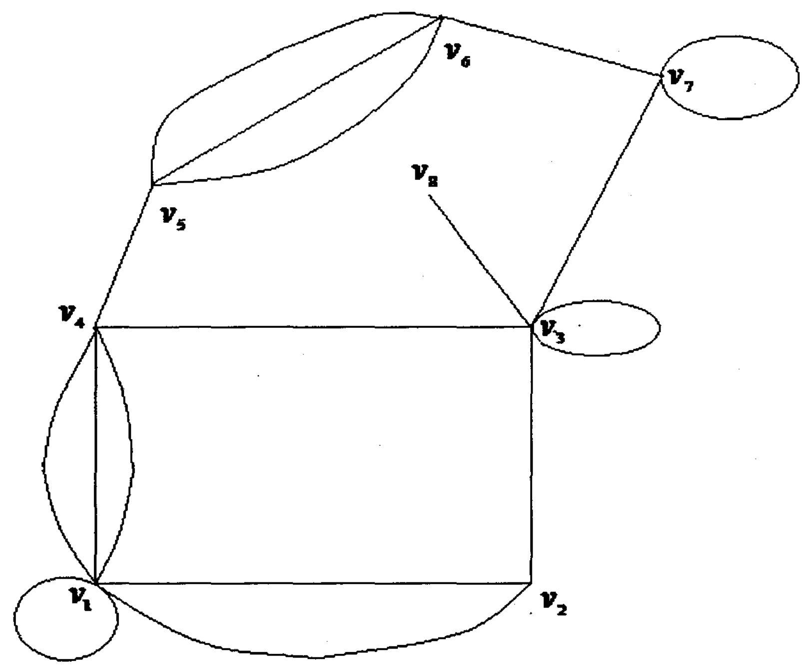
(b) Prove that there exists no simple graph with five vertices having degree 4, 4, 4, 2, 2.

(c) Show that any connected graph with n vertices and (n-1) edges is a tree.

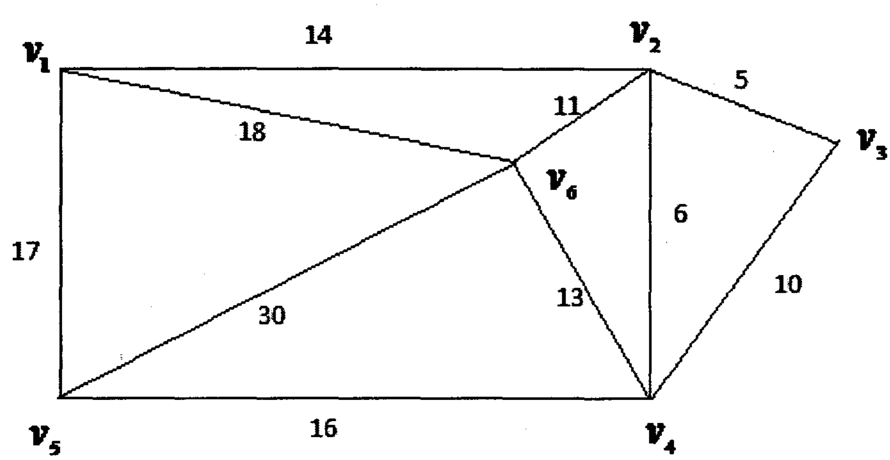
(d) Prove that for any connected planar graph with 'n' vertices, 'e' edges and 'f' regions, $n - e + f = 2$.

3+4+3+4

8 (a) Find the spanning tree of the graph.



(b) By Prim's algorithm find a minimal (shortest) spanning tree in the following graph and find the corresponding minimum weight.



6+8