M.E. (ETC) 1st Semester Final Examination, 2013-2014
Digital Processing & Control of Signal (ETC- 908)

Full Marks: 70 **Time: 3 hours**

Use separate answer scripts for each half

FIRST HALF

Answer Q. no.1 and any two from the rest

1. Discuss the problem of spectral analysis of non-stationary signals using Fourier transform. How does short time Fourier transform help to overcome the problem? Discuss its limitations. 4+7+4

2. Derive the relationship between discrete time Fourier transform (DTFT) and discrete Fourier transform (DFT) of a finite length, causal, discrete time signal. Is it possible to obtain a reliable frequency response of an infinite impulse response (IIR) filter using a digital computer? 6+4

3. a) Find the inverse Z-transform of the following function.

$$X(z)=\frac{3z^{-3}}{(1-\frac{1}{4}z^{-1})^2}, \quad x[n] \text{ left sided}$$

b) Determine the Z-transform of the following sequence

$$x[n]=\alpha^{|n|}, 0<|\alpha|<1$$

5+5

4. Write short notes on
   a. Linear convolution using DFT
   b. Energy compaction properties of DFT and discrete cosine transform (DCT)

5+5

SECOND HALF

Answer Q. No. 5 and any two from the rest

5. A random process $x(n)$ is generated as follows:

   $x(n) = \alpha x(n-1) + v(n) + \beta v(n-1)$; where $v(n)$ is white noise with mean m_v and variance σ_v^2.

   (a) Design a first-order linear predictor $\hat{x}(n+1) = w(0)x(n) + w(1)x(n-1)$ that minimizes the mean-square error in the prediction of $x(n+1)$, and find the minimum mean-square error.

   (b) Consider a predictor of the form $\hat{x}(n+1) = c + w(0)x(n) + w(1)x(n-1)$

Find the values for $c, w(0)$ and $w(1)$ those minimize the mean-square error, and compare the mean-square error of this predictor with that found in part (a).

Or

A signal $x(n)$ is observed in a noisy and reverberant environment, $y(n) = x(n) + 0.8x(n-1) + v(n)$; where $v(n)$ is white noise with variance $\sigma_v^2 = 1$ that is uncorrelated with $x(n)$. The signal $x(n)$ is a wide-sense stationary AR(1) process with autocorrelation values $\boldsymbol{r}_x = [4,\ 2,\ 1,\ 0.5]^T$. Find the non-causal IIR Wiener filter that produces the minimum mean-square estimate of $x(n)$ and also calculate the mean-square error. Derive all the expressions which you may use for your calculation.

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6. Show that for jointly wide-sense stationary processes, the steepest descent adaptive filter converges to the solution to the Wiener-Hopf equations if the step size satisfies the condition $0 < \mu < \frac{2}{\lambda_{max}}$; where λ_{max} is the maximum eigenvalue of the autocorrelation matrix $\boldsymbol{R}_x$.

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7. Examine the influence of zero-order hold on the continuous output responses $y(t)$ in Fig. A and Fig. B respectively by letting $G_1(s) = \frac{1-e^{-s\Delta t}}{s}$ and $G_2(s) = \frac{k}{s\tau+1}$. The input signal $x(t)$ is a unit step input.

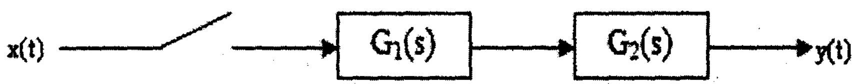

Fig. A

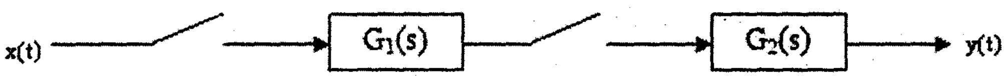

Fig. B

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8. Using the Jury stability criteria, determine if any roots of the characteristic equation $\Gamma(z) = 2z^4 - 3z^3 + 2z^2 - z + 1 = 0$ lies in the unstable region.

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