M.E. (ETC) 1st Semester Final Examination, 2012-2013
Digital Processing & Control of Signal (ETC- 908)

Full Marks: 70 **Time: 3 hours**

Use separate answer scripts for each half

FIRST HALF

Answer *Q. No. 1* and any *two* from the rest

1. (a)

For each of the pairs of sequences in Figure 1a, use discrete convolution to find the response to the input $x[n]$ of the linear time-invariant system with impulse response $h[n]$.

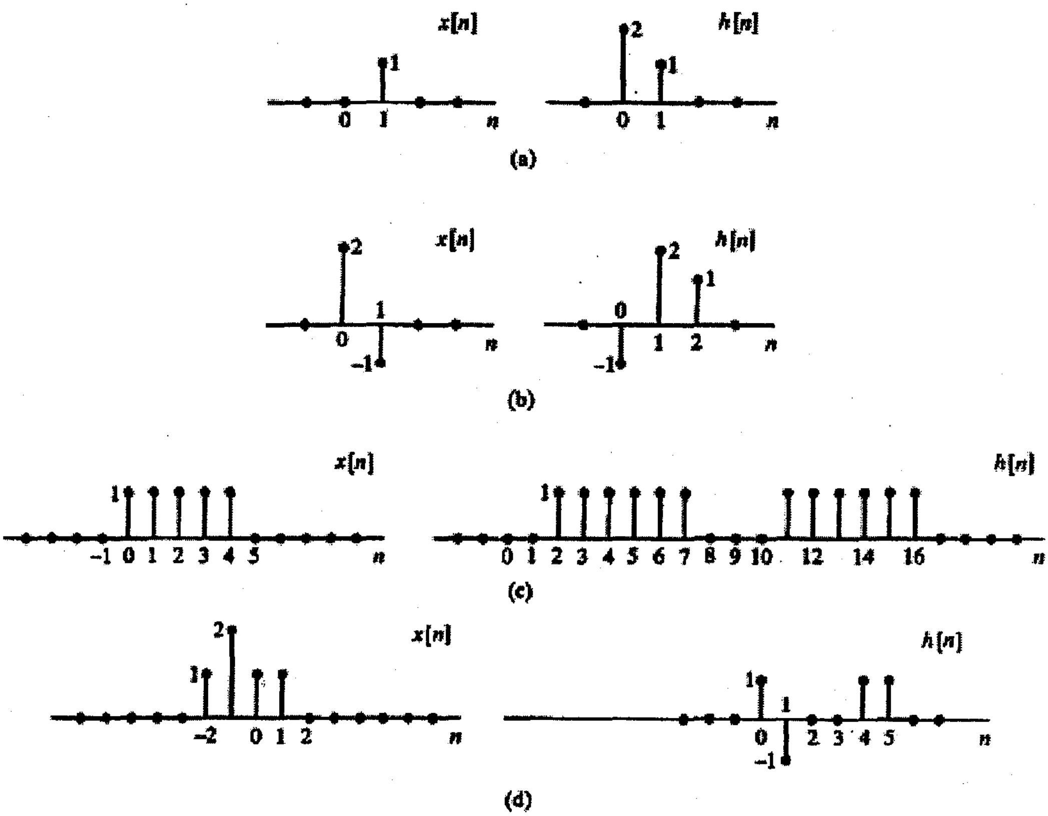

Figure 1a

(b)

The input–output pair shown in Figure 1b-1 is given for a stable LTI system.

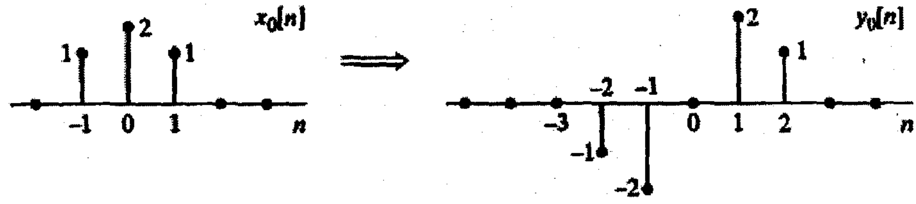

Figure 1b-1

Determine the response to the input $x_1[n]$ in Figure 1b-2

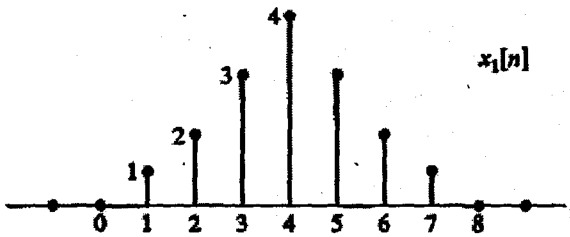

Figure 1b-2

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2. Determine the inverse Z-transform for the following:

$$X(z) = \frac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}, \qquad x[n] \text{ stable}$$

$$X(z) = \frac{3z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)^2}, \qquad x[n] \text{ left sided}$$

$$X(z) = e^{z^{-1}}$$

10

3. Describe the decimation in time FFT algorithm, drawing the signal flow graph for 16 point DFT in various stages.

10

4. Write notes on:
   (a) Structures for linear-phase FIR system
   (b) The JPEG standard

10

SECOND HALF

Answer *Q. No. 5* and any *two* from the rest

5. Consider a causal LTI system with system function $H(z) = \frac{1-a^{-1}z^{-1}}{1-az^{-1}}$, where 'a' is real.
   (a) Write the difference equation that relates the input and output of this system.
   (b) For what range of values of 'a' is this system stable?
   (c) For a=0.5, plot the pole-zero diagram and shade the ROC in z-plane.
   (d) Find the impulse response for this system.
   (e) Show that this system is an all-pass system. Also specify the value of the constant.

   Determine whether or not the discrete data system identified by the following characteristic equation is stable.

$$z^3 + 3.3z^2 + 4z + 0.8 = 0$$

10+5

6. Examine the influence of zero-order hold on the continuous output responses $y(t)$ in Fig. 5 (a) and Fig. 5 (b) respectively by letting $G_1(s) = \frac{1-e^{-s\Delta t}}{s}$ and $G_2(s) = \frac{k}{s\tau+1}$. The input signal $x(t)$ is a unit step input.

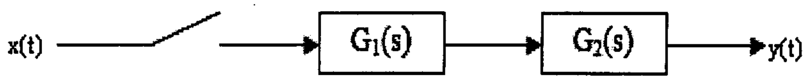

Fig. 5 (a)

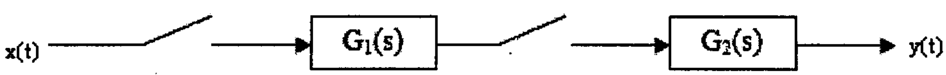

Fig. 5 (b)

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7. Find out the expression for delayed z-transform of the function $f(t) = e^{-at}u(t)$. Prove that modified z-transform of any function is not equal to its z-transform, unless the function is zero at time t=0.

5+5

8. Consider a system consisting of two sensors, each making a single measurement of an unknown constant x. Each measurement is noisy and may be modeled as follows:

$$y(1) = x + v(1)$$
$$y(2) = x + v(2)$$

   where $v(1)$ and $v(2)$ are zero-mean uncorrelated random variables with variances σ_1^2 and σ_2^2, respectively.
   (a) In the absence of any other information, the best linear estimate of x of the form $x' = k_1y(1) + k_2y(2)$ is to be found. Find the values for k_1 and k_2 that produce an unbiased estimate of x that minimizes the mean square error.
   (b) Repeat part (a) for the case where the measurement errors are correlated by $E\{v(1)v(2)\} = \rho\sigma_1\sigma_2$, where ρ is the correlation coefficient.

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