INDIAN INSTITUTE OF ENGINEERING SCIENCE & TECHNOLOGY, SHIBPUR
M.E. (ETCE) 2nd Semester Examination, 2014

Electromagnetic Metamaterials
(ETC-1038)

Time: 3 Hours

Full Marks: 70

Answer any FOUR Questions

1. (a) Draw the convergence and divergence of parallel beams in conventional right-hand (RH) lens. If this lens is replaced by a left-handed (LH) lens, what change will you observe in diverging and converging effects? Justify your answer by showing the details of propagation of three parallel rays (one in the middle and the two others on either side of the lens) at each interface between the lens and air.

   (b) Write the expression for the focal length of a lens in terms of radius of curvature of surface of lens (R) and refractive index (n). If n = +3 and n = –1, what would be the focal length in each case. From this, state the relevant advantage of LH lens.

   (c) Graphically illustrate the focusing of paraxial rays coming from a point source by a left-handed (LH) slab. Find the expression of the distance of outside focal point from the source point in terms of the refractive index and thickness of the slab.

2. (a) Write the Snell's law of refraction in terms of constitutive parameters or refractive index (n) of the media. From this law, explain the phenomenon of what happens when both ε and μ of a medium are negative simultaneously. In this case plot the transmitted ray along with the incident and reflected rays. Also sketch the figures when a straight stick is immersed in (i) empty glass, (ii) half-filled regular water and (iii) half-filled negative water (negative n).

   (b) Assume that the reflection coefficient is given by $\Gamma_{\perp} = \dfrac{\cos\theta_i - (\mu_1/\mu_2)\sqrt{n^2 - \sin^2\theta_i}}{\cos\theta_i + (\mu_1/\mu_2)\sqrt{n^2 - \sin^2\theta_i}}$

   where θ_i is the angle of incidence and n is the refractive index of the second medium with respect to the first one. From this equation explain in details whether the wave propagation is possible or not if either ε_2 or μ_2 is negative. Name two negative ε and two negative μ materials.

3. (a) Give the statements of four boundary conditions in case of conventional right-handed media. What change occurs to these statements if one of the two media is a left-handed medium? Give reasons.

   (b) Write the expression of phase velocity (v_p) and group velocity (v_g). Neatly plot ω-β diagram for waveguide propagation. How can you find these two velocities from ω-β diagram? Compare these velocities at different frequency of operation (below cut-off, above cut-off and tending to very high frequency) for waveguide propagation.

4. (a) Assume that at a fixed point in space E_x and E_y of a plane wave have equal amplitude and 180^0 time phase difference between them. Sketch the waveforms of these two components against time. Graphically plot the individual components and their resultant field at the instants $t = 0$, T/8, T/4, 3T/8, T/2, 5T/8 and find the locus of the tip of the resultant vector with time. Identify the type of polarization and justify your answer.

   (b) Consider two components E_x and E_y with unequal amplitudes of a plane wave and E_y is leading E_x by 90^0 in time phase. Write the expression of their resultant field in phasor form. From it find the corresponding instantaneous expression. Considering $z = 0$ as a fixed point in space and using the instantaneous expression, prove mathematically that the component waves forms an equation of an ellipse. Under what condition, this equation can be converted into an equation of a circle?

5. (a) Draw the equivalent circuit for a composite right left-handed (CRLH) transmission line. Write the expressions for the propagation constant, phase constant, guided wavelength, phase velocity (v_p) and group velocity (v_g). From these expressions, find v_p and v_g in terms of line parameters for pure left-handed (PLH) and pure right-handed (PRH) transmission lines. Plot v_p and v_g versus frequency in PLH and CRLH lines.

   (b) Write the values of α, β, v_p, v_g, λ_g, Z_c, μ, ε, n in the gap (stop band) of the dispersion curve of CRLH lines. Write these values at the transition frequency in case of balanced transmission. Plot Z_c vs. frequency in unbalanced and balanced transmission lines.

6. (a) Find the expression for loss tangent of a material and explain its significance with necessary vector diagram in characterizing the material.

   (b) Straight from the Maxwell's equation, derive the expression for complex permittivity and interpret each term of it. Relate the complex permittivity to loss tangent of a medium. With this complex permittivity find the expression for the intrinsic impedance (η) of a lossy medium from the expression of it for the perfect dielectric.

   (c) Assume that a plane wave is represented by equation $E = \cos(\omega t - \beta z)$. With the aid of neat sketches show that the constant phase point moves with a constant velocity. Find the expression of this velocity.

7. Write short notes on (any Two) :
   (a) Metamaterials are effectively homogeneous structure
   (b) Physical interpretation of $v_p \geq c$ in waveguide
   (c) Perpendicular polarization, parallel polarization
   (d) Axial ratio