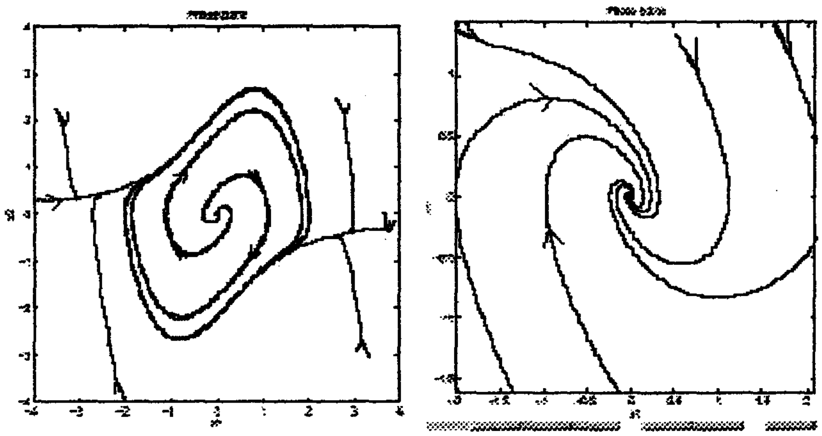
M.E. (E.E.) 1st semester Final, Examination , November, 2013
Subject : State Variable Analysis
EE 901

Full Marks: 70 **Time: 3 Hrs**

i) Answer any five questions taking at least one from Group A
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

1 (a) Two phase plots are given in the Fig. 1 (a) and (b). Match those two plots with the two equations given as (1a) and (1b) and justify your answer.



(a) (b)

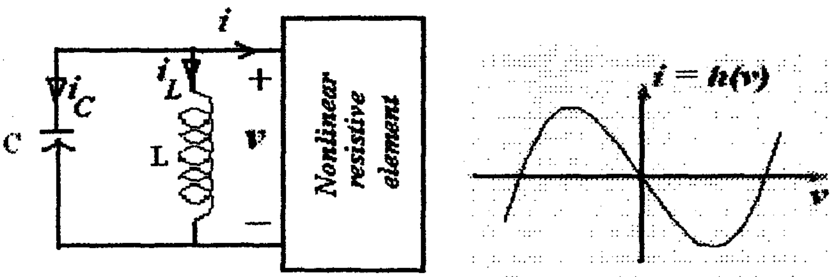
Fig. 1

$$(1a)\ \dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 - \sin(x_1)$$
$$(1b)\ \dot{x}_1 = x_2$$
$$\dot{x}_2 = (1 - x_1^2)x_2 - x_1$$

(b) A dynamic system is represented by the equation $\ddot{y} + \dot{y} + y - y^2 = 0$. Find the equilibrium points and sketch the phase plane near these equilibrium points.

[6+8]

2. A~~n~~ L_C circuit with nonlinear resistive element is shown in Fig. 2(a). The current entering the nonlinear resistive element is given by $i = h(v)$ where the function $h(v)$ is shown in Fig. 2(b).



(a) (b)

Fig. 2.

M.E. (E.E.) 1st semester Final, Examination, November, 2013
Subject : State Variable Analysis
EE 901

Full Marks: 70 **Time: 3 Hrs**

i) Answer any five questions taking at least one from Group A
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

(a) (i) Write the state space model of the system considering v and $\dot{v}$ are the state variable. (ii) Find the equilibrium point of the system and (iii) and find the nature of this equilibrium point.
(b) Justify that this system must have a limit cycle.

[(2+2+4)+6]

3. Equation of a pendulum with friction can be represented by the following equation

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\left(\frac{g}{l}\right)\sin x_1 - \left(\frac{k}{m}\right)x_2$$

(i) Find the equilibrium point and nature of the equilibrium point

(ii) Show that the motion will be asymptotically stable for any perturbation using Lyapunuv Direct Method.

[(2+3)+9]

Group B

Answers which are not to the point and brief will be penalised

4. (a) For the system:

$$\dot{X}(t) = A(t)X(t); X(t_0) = X_0$$

define the *state transition matrix* $\Phi(t,t_0)$.
Also show that:
i) $\Phi(t,t) = I$
ii) $\Phi(t_0,t) = \Phi^{-1}(t,t_0)$,
(b) What is the connection between *poles* of a system and the *eigenvalues* of the A matrix?
(c) The dynamics of a rocket is given by

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2}$$

And state variable feedback control is used where $x_1(t) = y(t); x_2(t) = \dot{y}(t)$ and $u(t) = -x_2(t) - 0.5x_1(t)$.
Determine the *time response* of the closed loop system when $x_1(0) = 0;\ x_2(t) = 1$.
Also find the *modes*.

M.E. (E.E.) 1st semester Final, Examination , November, 2013
Subject : State Variable Analysis
EE 901

Full Marks: 70 **Time: 3 Hrs**

i) Answer any five questions taking at least one from Group A
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

[6+2+6]

5. (a) Define *generalised eigen vectors* of a matrix. What is their importance?
(b). Convert the following system to its *diagonal* form

$$G(s) = \frac{10}{s(s+1)}$$

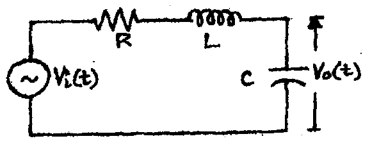
(c) What is the solution to:

$$\dot{X}(t) = A(t)X(t) + B(t)U(t); X(t_0) = 0$$

Justify your answer.

[4+6+4]

6. (a) For the system below make two different choices of state variables. How are these two different state vectors related?



(b) State the problem and the solution to the Linear Quadratic Regulator control problem.
(c) Find two different set of basis in $\Re^2$.

[6+4+4]

7. (a) What is Linear State Variable Feedback Control? What can it achieve?
(b) Express the system in 5 (b) in its Controllable Canonical Form.
Also design an LSVF controller to place poles at -2 ±j3.
c) State whether the system in 5 (b) is *reducible.* Justify without doing any calculations.

[4+7+3]

8. (a) Classify uncertainty giving examples.
(b) Write the equations of a full-order observer. State under what conditions does the estimation error decay to zero arbitrarily fast?
(c) Draw the block diagram and find an equivalent discrete state space representation if the system in 5 (b) is preceded by a sampler with sampling time 1 seconds and a zero order hold.

[4+4+6]