M.E. (EE) 1st semester Final, Examination , November, 2012
Subject : State Variable Analysis
EE 901

Full Marks: 70 **Time: 3 Hrs**

i) Answer any five questions taking at least one from Group A
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

Group A { At least one question from group-A}

1. (a) What is a vector field in a phase plane?, (b) Find the equation of the phase trajectory of the system $\ddot{\theta}+\omega^2\sin\theta=0$ analytically, (c) Sketch the trajectory of the motion of the pendulum for an initial condition of $\theta=0$ and $\dot{\theta}=2\omega$. (d) Will the trajectory cross the θ axis? Justify your answer. [2+4+5+3]

2. (a) Find the equilibrium point of the system $\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1-\varepsilon x_1^2 x_2\end{aligned}$, (b) Examine the small perturbation stability of the system around the equilibrium point, (c) Sketch the phase portraits near the equilibrium point for $\varepsilon<0$. (d) What is a limit cycle? (e) Establish that the system $\ddot{v}-\varepsilon(1-v^2)\dot{v}+v=0$ has a limit cycle.

[2+4+2+2+4]

3. (a) State Lyapunov's second or direct method to examine the stability at large.
(b)Determine the stability of the system $\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -g(x_1)+x_2\end{aligned}$ around its equilibrium point at origin. Given that $\dfrac{g(x_1)}{x_1}>0$ for $x_1\neq 0$

[6+8]

Group B

4. (a) Find a *state space model* for the system in Fig. 1. Hence find the *time response* of the system for a unit step function applied at "t=0" and zero initial conditions.
(b) Define a *fundamental matrix* for:

$$\dot{X}(t)=A(t)X(t);X(t_0)=X_0$$

Hence define the *state transition matrix* for the above system.

[5+6+3]

5. (a) Define the *impulse response* function. How can we examine the *stability* of a Linear Multivariable MIMO system given by:

$$\dot{X}(t)=A(t)X(t)+B(t)U(t);X(t_0)=0$$

(b). Design a *Linear State Variable Feedback* controller for the system:

$$G(s)=\frac{10}{s(s+1)}$$

to place the poles at (-1±j1).

[4+10]

6. (a) Are the following vectors *linearly independent*?

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$$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}-1\\0\\2\end{bmatrix}, \begin{bmatrix}0\\2\\5\end{bmatrix}$$

(b) State the importance of the loop *Sensitivity function* in control system design.
(c) "All *eigen values* may not appear as *poles*"- justify.

[7+3+4]

7. (a) What is an Observer? How does the estimation error go to zero?
(b) Design a state observer for the system:

$$\dot{X}(t) = \begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix}X(t) + \begin{bmatrix}1\\-1\end{bmatrix}U(t);$$
$$Y(t) = \begin{bmatrix}1 & 0\end{bmatrix}$$

Such that the estimation error decays in less than 4 secs.

[4+10]

8. (a) Examine whether the system in 7 (b) is reducible or not:

(b) Find the state transition matrix of:

$$\dot{X}(t) = \begin{bmatrix}1 & 0\\e^{-t} & 0\end{bmatrix}X(t)$$

[7+7]

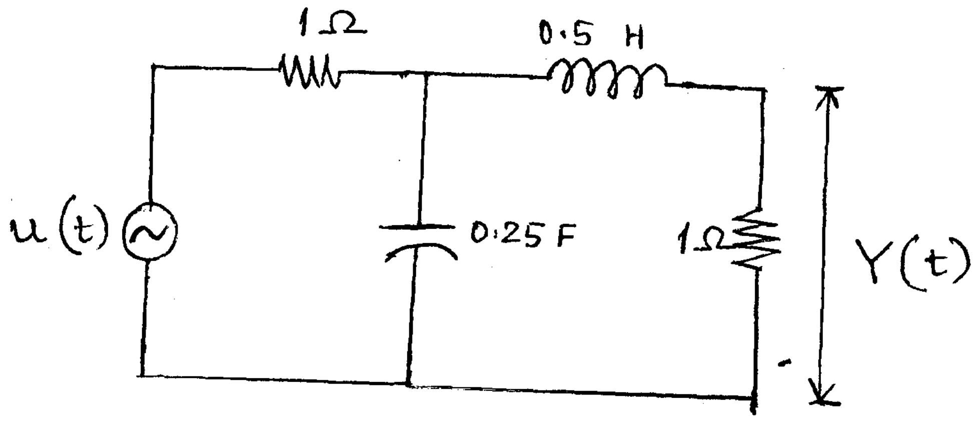

Fig. 1