M.E. (Elec.) 2nd Semester Examination, 2014

Optimal Filtering Process (EE-1022)

Time : 3 Hours | Full Marks: 70

Answer any FIVE questions

1. a) What are *phenomena* and *noumena*? What causes the differences between phenomena and noumena? How optimal filtering process helps in control of stochastic processes? What are the application areas of Optimal Filtering?

   b) Discuss on the computational origin of Kalman Filter and hence give the physical significances of "Time update" and "Measurement update" equations showing the block diagram of the recursive process. [7 + 7]

2. a) What form of Kalman filter algorithm would you prefer in case there is a large error in initial guess of the states? How could this method eliminate the effect of large initial error – explain with the help of sequence diagram and update equations.

   b) Comment on the stability condition of the following discrete-time system,

   $$x_{k+1} = 2x_k + w_k$$
   $$z_k = x_k + v_k$$

   , when the steady state gain of the filter approaches 0.8. [9 + 5]

3. a) What are the assumptions for applying Kalman filter in nonlinear systems? How do you find the Jacobians for nonlinear systems? Write down the equations of discrete time Extended Kalman Filter.

   b) Find the state estimates of the following nonlinear process using the Discrete Linearized Kalman Filter equations up to $k = 3$.

   $$x_{k+1} = 2x_k^{\,3} + w_k$$
   $$z_k = x_k^{\,2} + v_k$$
   $$E\langle w_k \rangle = E\langle v_k \rangle = 0$$
   $$E\langle x_0 \rangle = x_k^{NOM} = 2$$
   $$P_0 = 1$$
   $$\text{cov.}(w_k) = 2\Delta(k_2 - k_1)$$
   $$\text{cov.}(v_k) = \Delta(k_2 - k_1)$$
   $$z_1 = 1,\ z_2 = 1.3,\ z_3 = 1.2$$

   [8 + 6]

4. a) Define the terms 'terminal penalty cost', 'state weighting matrix' and 'input weighting matrix' in the control of stochastic processes. How these terms are selected for design of such control? Draw the block diagram of a LQG control system for a tracking problem and write the series of equations required to be solved.

   b) How the LQG problem in a stochastic process can be separated into two completely decoupled processes of optimal estimation and optimal control? Explain with relevant matrix equations. [9 + 5]

5. a) What are the causes of unpredicted non-convergence of the Kalman filters? What causes the modeling problem? How this modeling problem can be handled in optimal filtering process?

   b) How periodic disturbances can be modeled? Derive an augmented system with modeling the process disturbance only.

   c) An Inventory model consists of the following components. $y(k)$ is the level of the inventory at time (day) k; $u_1(k)$ is the orders placed for day k; ordered items takes two days for delivery and $u_2(k)$ is the sales for day k. Write the difference equation of the system and form the state-space equation with one output (inventory stock) and two inputs (order placed and sales). If sales is considered as a disturbance (unpredictable), i.e. $w(k) = u_2(k)$, and the sales on consecutive days are positively correlated as $u_2(k) = v(k) = w(k) + 0.9w(k-1)$ and $E\{v(k).v(k+1)\}/E\{v^2(k)\} = 0.9/1.81$. What will be the output equation of the augmented system? [5 + 4 + 5]

6. a) Define the following –

      i) Random Variables - discrete & continuous

      ii) Probability Density Function of Random variables.

      iii) Functions of Random Variables

   b) Consider choosing a card from a well-shuffled standard deck of 52 playing cards. What is the probability that the first card extracted is an ace? If after the first extraction, the card is not reinserted in the deck, what is the probability that the second card is an ace, given that the first card is an ace? Let A be the event that the first card is an ace, and let B be the event that the second card is an ace. Find the conditional probability of A|B using:

$$p(B|A) = \frac{p(B,A)}{p(A)}.$$

[9 + 5]

7. a) Express the Expectation and Variance of linear combination of random variables.

   b) Suppose we roll two die and take their sum S = {2, 3, 4, 5, .., 11, 12}. Find the probability that the sum=5 and show the combinations.

   c) A company manufactures steel bars of nominal diameter 20mm and cuts them in equal length of bars. Diameter of each bar is measured at the middle and the data for a lot of 20 such specimens are: 19.9, 19.8, 20.1, 19.9, 19.7, 20.1, 20.0, 19.6, 19.7, 20.1, 20.2, 20.0, 19.9, 19.8, 20.1, 20.0, 19.6, 19.7, 19.9 and 20.2. Determine the mean, median, mode, variance and standard deviation. [4 + 4 + 6]

8. Write critical notes on ***any two***: [2 x 7 = 14]

   a) Bad data problem and its recovery.

   b) Loop transfer recovery in LQG process.

   c) Kalman filter and deterministic least square method.

   d) Sigma point computation for unscented filter.

--- x ---