M.E. (Elec.) 2nd Semester Examination, 2013

Optimal Filtering Process (EE-1022)

Time : 3 Hours

Full Marks: 70

Answer any FIVE questions taking Two from Gr-A and Three from Gr-B
1 mark reserved for neatness

Group-A

(Answer any **Two** Questions. [12x2=24])

1. a) What are 'time varying' and 'time invariant' systems? Explain 'observability' of a time invariant system.

   b) Discuss the following–

      i) Expected values of Random Variables

      ii) Functions of Random variables. [6+6]

2. a) Discuss the problem of tossing a die and two dice in terms of random variables.

   b) State and explain auto-correlation function with the help of an example. [6+6]

3. a) What is a Wiener filter? Compare its operation with that of Kalman filter

   b) Discuss the differences among 'predicter', 'filter' and 'smoother'. [6+6]

Group-B

(Answer any **Three** Questions. [15x3=45])

4. a) What is a Kalman filter? Give the computational origin of a Kalman filter and hence establish the "Time-update" and "Measurement-update" equations. Draw the block diagram of the recursive process of Kalman filter.

   b) How deterministic least square method may be related with a Kalman filter algorithm? [10+5]

5. a) Discuss the linearization methods in optimal filtering process? Write down the equations/algorithms for the LKF and EKF.

   b) Find the LKF estimations up to $k=3$ for the following system with $z_1=1$, $z_2=3$, $z_3=2$:

$$x_{k+1} = -0.5x_k^2 + x_k + w_k$$

$$z_k = x_k^{\;3} + v_k$$

$$E\langle w_k \rangle = E\langle v_k \rangle = E\langle x_0 \rangle = x_0 = 0$$

$$\text{cov.}(w_k) = 2\Delta(k_2 - k_1)$$

$$\text{cov.}(v_k) = 0.5\Delta(k_2 - k_1)$$

$$P_0 = 1; \quad x_k^{NOM} = 2$$

[9+6]

6. a) Define the terms 'terminal penalty cost', 'state weighting matrix' and 'input weighting matrix' in the control of stochastic processes. How these terms are selected for design of such control? Draw the block diagram of a LQG control system for a tracking problem and write the chain of equations required to be solved.

   b) How the LQG problem in a stochastic system can be decoupled into two separate processes of optimal estimation problem and optimal control problem? [10+5]

7. a) What causes the unpredicted non-convergence in Kalman Filters? Give some of the remedies of Bad-data problem.

   b) What is a modeling problem? How this modeling problem can be handled by augmenting the states?

   c) A heat exchanger system is given by the following state-space representation;

$$\dot{x} = \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d$$

$$z = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

   u is the control input and d is the process disturbance of external temperature having a periodic variation given by $\dot{d} + 0.001d = w$, where w is a white noise process. The sensor noise is neglected. How would you model the disturbance to augment the unmodeled states of the system?

[6+3+6]

8. Write critical notes on ***<u>any three</u>***: [3x 5]

   a) Round-off error problem.

   b) Stability aspect in a closed loop system with a Kalman filter

   c) Alternative form of Kalman filter

   d) Sigma point computation for unscented filter.

   e) Discrete time LQG control problem.