M.E. (EE) 2nd semester Final, Examination, May, 2014
Subject : Optimal Control I
EE 1002

Full Marks: 70 **Time: 3 Hrs**

i) Answer any FIVE questions
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

1. a) In Calculus of Variations starting from Euler's equations, derive the necessary conditions obeyed by a function $y(x)$ in order to attain a minimum for the functional,

$J = \int_{x_1}^{x_2} F\{x; y(x); y'(x)\}dx$, when x_1 and x_2 may vary. Clearly state the assumptions made during the derivation.

b) Minimise $J = \int_{x_0}^{x_1} \{3x + \sqrt{y'}\}dx$; for a choice of $y(x)$ symbols having their usual significance.

[10+10]

2. a) Maximise $J_1 = \int_{-a}^{a} \{y(x)\}dx$, subject to constraints:

$J_2 = \int_{-a}^{a} \sqrt{\{1 + y'^2(x)\}}dx = L$ and $y(-a) = y(a) = 0$

b) A D. C. motor is used for excavation load. Assuming the initial and final angular velocities to be zero, obtain the best angular velocity and current profiles such that the total angle traversed in a given time is maximized while the losses remain constant.

[10+10]

3. a) In the linear quadratic regulator problem, assuming the expression of the optimal input, work out the minimum value of the performance index.

b) Find $u(t)$ that minimizes $J = \int_{0}^{\infty} \{x^2(t) + u^2(t)\}dt$ subject to $\dot{x}(t) = -x(t) + u(t)$.

[10+10]

4. a) Establish the robustness properties of the LQR control, assuming the Return Difference Inequality.

b) State and justify the condition for achieving Robust Stabilty in the H∞ control problem.

[13+12]

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5. a) State the Schur complement Lemma.

b) Write down the condition for Lyapunov stability of a linear time invariant autonomous state equation. Show how it can be converted to an LMI.

c) State is the condition for the expression below to be convex for any vector x.

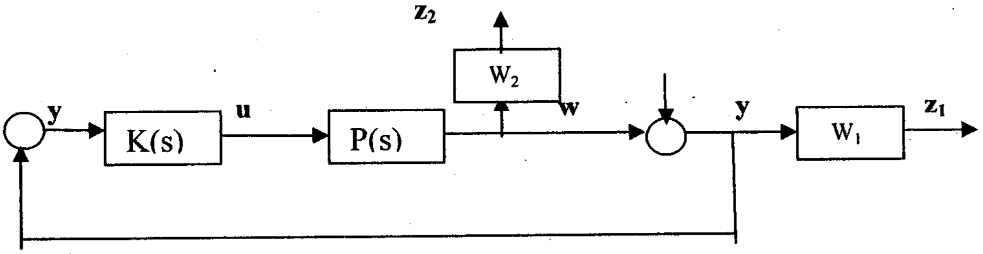
$$x^T P x$$

[5+10+5]

6. a) During minimising the function: $f(X) = x_1 + 0.5x_2 + 4x_1^2 + 2x_2^2$ using Cauchy's Steepest Descent Method, show the first two steps of iterations.

b) Mention one criterion which may be used for the Convergence of the algorithm.

c) For the block diagram below, formulate the standard problem in H∞ optimal control theory.



Also calculate the cost transfer function to be minimized.

d) Define the ||. ||∞ norm of scalar and a multivariable transfer function.

[10+3+5+2]

7. a) What is an LMI? Give an example.

b) Test whether the following function is Convex or not:

$$f(X) = 3x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3 - 6x_1 - 4x_2 - 2x_3$$

c) What is the geometric representation of the equation below:

$y = \theta(x_1) + (1 - \theta)\, x_2$, where $x_1, x_2 \in \mathbb{R}^2$ and $x_1 \neq x_1$

[4+10+6]