M.E. (EE) 2nd semester Final, Examination , April, 2013
Subject : Optimal Control I
EE 1002

Full Marks: 70 **Time: 3 Hrs**

i) Answer any four questions
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

1. a) In Calculus of Variations derive the necessary conditions obeyed by a function $y(x)$ in order to attain a minimum for the functional, $J = \int_a^b F\{x; y(x); y'(x)\}dx$. Clearly state the assumptions made.

b) Minimise $J = \int_{x_0}^{x_1} \{x^2 + x^2(y'^2)\}dx$; for a choice of $y(x)$.

[16+9]

2. a) Starting from the standard dynamic equations (including inertia) of a d. c. motor derive a dimensionless dynamic equation of the motor relating the angular velocity of the motor and the armature current.
b) A D. C. motor is used for excavation load. Assuming the initial and final angular velocities to be zero, obtain the best angular velocity and current profiles such that the total angle traversed in a given time is maximized while the losses remains constant.

[7+18]

3. a) In the linear quadratic regulator problem, derive an expression of the optimal input in terms of the adjoint variable. Also write the equations of the closed loop system.

b). Find $u(t)$ that minimizes $J = \int_0^2 \{x^2(t) + u^2(t)\}dt$ subject to $\dot{x}(t) = -x(t) + u(t)$.

[15+10]

4. a) State the solution to a "constrained optimization problem".

b) Minimise $J = \int_0^{\pi} \{y'^2(x) - y^2(x)\}dx$, where, $y(0) = 0,\ y(\pi) = 1$.

c) What is Legendre's condition? Where is it used?

[5+10+10]

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5. a) In the LQR problem, derive the Return Difference Inequality. Mention in brief its physical significance
b) Obtain the Control Law which minimizes the performance index:

$$J = \int_0^\infty \{x_1^2(t) + u^2(t)\}dt; \text{ for the system described by:}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

[13+12]

6. a) Minimise the function: $f(X) = x_1 + 0.5x_2 + 4x_1^2 + 2x_2^2$ using the Conjugate Gradient Method.
b) Express the following function: $f(X) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ as a standard quadratic function. Then check whether the following search directions are [A]-conjugate or not:

i) $S_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}; S_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,

ii) $S_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}; S_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

c) Mention one criteria which may be used for the Convergence of optimization algorithms.

[10+6+9]

7. a) What is an LMI? Give an example.
b) Test whether the following function is Convex or not:
$f(X) = 3x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3 - 6x_1 - 4x_2 - 2x_3$
c) Explain the geometric significance of the following:

$y = \theta(x_1) + (1 - \theta)\, x_2$, where $x_1, x_2 \in \mathbb{R}^2$ and $x_1 \neq x_1$
d) Define a convex function with a sketch. What is a convex optimization problem?

[6+6+7+6]