

Full Marks: 100

Time: 3 Hrs

- i) Answer any four questions
- ii) Questions are of equal value
- iii) Credit will be given to brief and to the point answers

1. a) In Calculus of Variations derive the necessary conditions obeyed by a function $y(x)$ in order to attain an extremum for the functional, $J = \int_a^b F\{x; y(x); y'(x)\} dx$, symbols having their usual significance. What are the assumptions made?

b) When is a matrix positive definite? Test whether the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ is positive definite or not.

[19+6]

2. a) Find the optimum current and velocity profiles of a separately excited d. c. motor, used for excavation loads, in a way such that the armature losses are kept constant while the total angle traversed in a given time is maximized. Assume initial and final velocities to be zero.

b) What is a convex optimization problem? What is a "feasible set"?

[19+6]

3. a) Derive an expression of the optimal control input in the Linear Quadratic Regulator control in terms of the adjoint variable.

b) Check whether the following functions are convex or not:

i) X^{-1} for $X \in \mathfrak{R}_{++}$, and ii) e^{aX} on \mathfrak{R} , for any $a \in \mathfrak{R}, X \in \mathfrak{R}$.

[15+10]

4.a) Determine the optimal curves $y(x)$ and $z(x)$ for the performance index:

$$J = \int_0^{\frac{\pi}{2}} \{\dot{y}^2(x) + \dot{z}^2(x) + 2y(x)z(x)\} dx \quad \text{subject to } y(0) = 0; z(0) = 0; y\left(\frac{\pi}{2}\right) = -1; z\left(\frac{\pi}{2}\right) = 1.$$

b) With an example show that the problem of 'regulating' any arbitrary state to a desired nonzero state may be cast as a problem of bringing the nonzero states to zero.

[20+5]

M.E. (EE) 2nd semester Final, Examination , 2012
Subject : Optimal Control Systems- I
EE 1002

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5. a) Write the dynamic equations of the closed loop system under Linear Quadratic Regulator control.

b) Examine the extremums of the functional:

$$J = \int_0^2 (xy' + y'^2) dx, \text{ subject to } y(0) = 0; y(2) = 1.$$

[5+20]

6. a) Briefly explain the significance of the Sensitivity function in a SISO feedback loop.

b) Using Conjugate Gradient Method, find the minimum of the function:

$$f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 4x_1x_2 + x_1, \text{ from the starting point } \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

[8+17]

7. a) State the properties of the Linear Time Invariant LQR control.

b) Classify 'Uncertainty', giving examples.

c) What is an 'LMI'? When is it 'feasible'?

[12+7+6]