

M.E. 2nd Semester (EE) Final Examination, 2014
Non-Linear Control Systems
(EE-1001)

Answer any FIVE questions

Questions are of equal values

Graph papers will be supplied, on demand.

Time : 3 hours

Full Marks : 70

- 1(a) Deduce nonlinear differential equations that define the dynamic behavior of an orbiting satellite along with the state space representation of the system.
- (b) Linearize the above system with satellite mass M considered to be unity. Verify that the rotation of the satellite at a constant radial distance $d=r(t)$ and at a constant angular velocity ω represents a particular nominal solution that can be linearised around.
- (c) What is 'Vibrational Linearization'? Explain with diagram how the response of a dead zone type element changes when additional periodic signal is introduced.

[5+5+4]

2(a) What do you mean by perturbation method?

- (b) Consider the case of a heavily damped simple pendulum. Find the solution of such system using Poincare perturbation method.

(c) Explain 'secular term'.

[3+8+3]

3(a) Consider the following second order nonlinear system as follows

$$\ddot{x} + F(x, \dot{x}) = 0$$

Use Krylov and Bogoliubov method to solve the above system.

- (b) Solve the following first order equation using 4th order Runge Kutta method with $h=0.05$ and $y(0.05)=0.4423$ upto x value = 0.25.

$$\frac{dy}{dx} = -xy - x - y$$

Comment on the convergence of the result.

[7+7]

4(a) Differentiate between linear and non-linear systems.

(b) Find the solution and draw the phase portrait of the linear system, $\dot{x} = Ax$, where

$$\dot{x}_1 = -x_1 - 3x_2$$

$$\dot{x}_2 = 2x_2$$

(c) State and explain the Diagonalization theorem of an $n \times n$ matrix A is given by $\dot{x}_1 = Ax$, to an uncoupled linear system

[2+6+6]

5(a) Solve the initial value problem of $\dot{x}_1 = Ax$ with initial point $x(0)$ and A are respectively given below.

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix} \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(b) Explain clearly with example of the following :

- i) A saddle node at the origin
- ii) A stable node at the origin
- iii) A stable focus at the origin

[8+6]

6(a) Use delta method and construct the phase-trajectory of the system defined by the equation

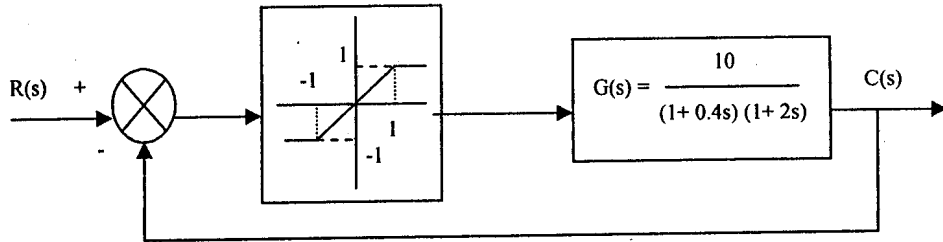
$$dx_2/dx_1 = -[x_1 + (0.58x_1^3 + 0.33x_2)] / x_2$$

(b) Define the various sign of definiteness. Example them with examples [7+7]

7. Determine the amplitude and frequency of the limit cycle of the non-linearity shown in Fig.-1 by drawing the followings:

- (i) derive the describing function of the non-linear element
- (ii) draw the characteristic of the describing function
- (iii) draw the characteristic of the plant transfer function of which is $G(s)$

[4+(4+3+3)]



Q.-7 : Fig.-1

8(a) The state equation for a linear system are given by $\dot{x} = Ax$, where

$$A = \begin{bmatrix} -5 & 1 & -2 \\ 1 & 0 & -1 \\ 3 & 2 & 4 \end{bmatrix}$$

Investigate the stability of the system by finding a suitable Lyapunov function. [8]

(b) Discuss the Popov's criterion for stability of non-linear systems. [6]