

Time : 3 hours

Full Marks: 70

Answer any FIVE Questions
All Questions carry equal Marks

1. a) What are the sufficient conditions of admissible transformation. Derive Jacobian matrix using admissible transformation.
b) Derive stress invariants using tensor notation.
c) Show that $\delta_{ij} \cdot \delta_{ji} = 3$

(7+5+2=14)

2. a) Define and explain octahedral shear stress.

c) The given state of stress is: $\sigma_x = 12.31$, $\sigma_y = 8.96$, $\sigma_z = 4.34$, $\tau_{xy} = 4.2$, $\tau_{yz} = 5.29$, $\tau_{zx} = 0.84$. Calculate principal stresses and direction of maximum principal stress. Also compute octahedral shear stress.

(4+10=14)

3. a) Derive Lamé's displacement equations of equilibrium.

b) Explain compatibility condition.

c) For the following strain distribution, verify whether the compatibility condition is satisfied

$$\varepsilon_x = 3x^2y \quad \varepsilon_y = 4y^2x + 10^{-2} \quad \gamma_{xy} = 2xy + 2x^3 \sigma_y$$

(8+2+4=14)

4. Derive the following differential equations of equilibrium in the polar co-ordinate system.

$$\frac{\sigma_r - \sigma_\theta}{r} + \frac{\Delta\sigma_r}{\Delta r} + \frac{1}{r} \frac{\Delta\tau_{r\theta}}{\Delta\theta} + R = 0$$

$$\text{and } \frac{1}{r} \frac{\Delta\sigma_\theta}{\Delta\theta} + \frac{\Delta\tau_{r\theta}}{\Delta r} + \frac{2\tau_{r\theta}}{r} + S = 0$$

(14)

5. a) Explain what is membrane analogy for torsion problems and also derive the expressions for a bar of thin rectangular cross-section due to torsion.

b) A tube of outside diameter 120mm and inside diameter 100mm is subjected to a torque of 10 kN-m. Find the maximum shear, tensile and compressive stresses in the tube. Show these stresses on properly oriented stress elements.

(7+7 = 14)

6.a) What do you mean by a yield criterion? Explain Trescas and Von Mises yield criteria. Also, give the graphical representations.

b) A spherical shell of inner radius 'a' and outer radius 'b' is subjected to internal pressure 'p'. Assuming perfect plasticity and Von Mises yield criterion find the value of 'p' at which yielding just starts. How will the result differ if Trescas criterion is used? Also obtain the final value of the pressure for which the whole shell becomes plastic.

(7+7 = 14)

7. What do you mean by 'equations of plasticity'? Explain Mises equations of plasticity and Prandtl-Rouss equations of plasticity, clearly stating the assumptions.

(3+4+7 = 14)

8. Derive an expression relating torsion and twist in a circular shaft stating the assumptions and explain what you mean by torsion constant. Hence, derive expressions for maximum shear stress for an elliptical shaft subjected to torsion T.

Where a & b are the major and minor axes of the ellipse.

(9+5= 14)
