

# Bengal Engineering and Science University, Shibpur

M. E. (Civil) 2<sup>nd</sup> Semester Final Examination, April 2013

Full Marks: 70

Time 3 hours

## Subject: Probabilistic Design of Structures (CE 1026)

Answer any Five Questions. All questions are of equal value. Assume any data reasonably, if required. All the notations used have their usual meanings.

1. A simply supported beam is of span 5 m and the dead load is assumed as distributed load,  $w$ , the concentrated live load,  $P$  is applied at the centre, the moment capacity of the beam is  $M_R$  and all these parameters are uncorrelated random variables. The distribution parameters for the random variables are given as: Distributed load, expected value,  $\mu_w = 20$  kN/m, cov,  $V_w = 10\%$ , Concentrated load, expected value,  $\mu_P = 50$  kN, cov,  $V_P = 15\%$  and Moment capacity, expected value,  $\mu_{MR} = 100$  kNm, cov,  $V_{MR} = 10\%$ . Calculate the reliability index,  $\beta$  by FOSM.

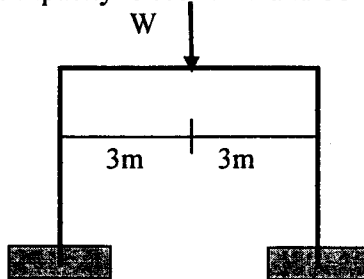
2. A tension member is subjected to an axial load,  $P$ . The allowable tensile stress is  $R$  and the diameter of the circular cross section is  $d$ . All these parameters are uncorrelated normal random variables with following given data: Load, expected value,  $\mu_P = 5000$  N, standard deviation,  $\sigma_P = 2000$  N Allowable stress,  $\mu_R = 280$  N/mm<sup>2</sup>, standard deviation,  $\sigma_R = 28$  N/mm<sup>2</sup> Diameter,  $\mu_d = 6$  mm, standard deviation,  $\sigma_d = 0.6$  mm. Find the reliability index by FORM algorithm.

3. A steel beam having length of 8m is subjected to a concentrated load at centre. The allowable yield strength is  $\sigma_y$ , plastic moment section modulus is  $Z_p$ . The mean value and standard deviations of various random parameters are as following:

	Mean	Nominal	Variance
P	10 kN	12kN	4
$\sigma_y$	$600 \times 10^3$ kN/m <sup>2</sup>	$550 \times 10^3$	$100 \times 10^6$

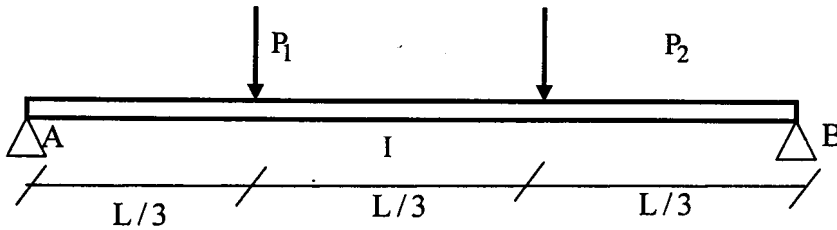
Obtain the partial safety factors for target reliability index of 3.0

4. Compute the system reliability of the portal shown in Fig. with following given data assuming all random variables are uncorrelated Gaussian:  $\mu_w = 446$  kN,  $\sigma_w = 69.9$  kN. For columns, the mean allowable plastic moment capacity is 490 kn-m and SD is 73.5 kN-m. For beam, the mean allowable plastic moment capacity is 653 kn-m and SD= 97.95 kN-m.



5. Find reliability index of the beam (shown in the figure) in flexure. Given Data:

$\mu_{P_1} = 10\text{kN}$ ,  $\sigma_{P_1} = 2\text{kN}$ ,  $\mu_{P_2} = 15\text{kN}$  and  $\sigma_{P_2} = 3\text{kN}$  and correlation coefficient between  $P_1$  and  $P_2$ ,  $\rho_{12} = 0.25$  Allowable moment,  $\mu_M = 50\text{kN-m}$  and  $\sigma_M = 5\text{kN-m}$ , Length,  $L=9\text{m}$ ,



(b) Explain how correlated normal random variables can be tackled in FORM algorithm for non-linear limit state equation.

6. Write short notes on any *four* of the followings:

i) FOSM with its limitations, ii) Determination of variance of  $y=g(x_i)$ , mean and variance of  $x_i$ 's are given, iii) Reliability Bounds for series system, iv) Reduction of insensitive random variables by Response Surface Method, v) Application of Beys theorem in Structural Engineering, vi) Rosenblueth's  $2k+1$  point estimate method

7. a) A structure may be subjected to dead load (D), live load (L), wind load (W) and seismic load (S).  $P(L)=0.8$ ,  $P(W)=0.4$ ,  $P(S)=0.15$ ,

i) represent the loads using Venn Diagram,

ii)  $P(D)=?$

iii) Calculate  $P(DL)$ .

iv) Calculate  $P(LW)$ ,  $P(DLW)$  and  $P(LS)$

v) Calculate  $P(D \cup L \cup W)$  and  $P(D \cup L \cup S)$ .

b) A pile foundation can be damaged due to failure in pile cap (C) or pile (P) if soil does not fail. Corresponding failure probabilities are  $P(C)=0.02$ ,  $P(P)=0.06$ . If there is pile failure probability that the cap will also suffer some damage is 0.3. What is the probability of failure of the pile foundation system? What is the probability of failure of pile if there is a pile cap failure?

c) A steel cable has to carry a weight of 150 kN. Information on the strength of similar cable indicates that the strength of the cable can be modeled by a normal random variable with a mean of 140 kN and a standard deviation of 30 kN. Calculate the probability of failure of the cable.

8. a) What are the advantages and disadvantages of Monte Carlo Simulation?

b) Using Latin Hypercube Sampling with five divisions calculate mean and coefficient of variation of  $y=17x_1-x_2^2$ , where  $x_1$  is uniformly distributed between 5 to 20 and  $x_2$  is normally distributed with mean 10 and standard deviation 0.15. Also, estimate the CDF of y.

9) Show the design of experiment and design matrix for type-III  $2^k$  FD model with two variables.

b) Obtain a type -I response surface model for  $I_1 = 0.34\alpha_s D^3 t_{cf}$ , where  $\alpha_s = 30 / (kL/r_y)^2$  as a function of  $kL/r_y$ .  $D=1200$  mm,  $t_{cf}=36$  mm. Take mean value of  $kL/r_y$  as 20.

### A.3 STANDARDIZED NORMAL DISTRIBUTION FUNCTION

Table A.3 Standardized normal distribution function: a table of

$$F_U(u) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^u e^{-x^2/2} dx,$$

for  $u = 0.0$  to  $3.69$

$u$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5733
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9482	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.8874	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

From Parzen, E., 1960, *Modern Probability and Its Applications*, John Wiley & Sons, with permission.