M.E. (Civil) 2nd Semester Examination, April-May, 2013

THEORETICAL SOIL MECHANICS (CE 1011)

Time: 3 hours Full Marks: 100

Answer any five questions

1.(a) Derive the matrix equation $\mathbf{S_1} = \mathbf{ASA^T}$ relating the change in stress components under the state of rotation of axis system about the origin.
(b) Derive the differential equations of equilibrium in three dimensions (2x10)

2.(a) For determination of the principal stresses at a point in a soil mass, derive the characteristic equation $\sigma_n^3 - I_1 \sigma_n^2 + I_2 \sigma_n - I_3 = 0$
(b) Why are I_1, I_2, and I_3 called invariants?
(c) Show that the principal stresses are mutually orthogonal. (8+2+10)

3.(a) Derive expressions for the octahedral normal and shear stresses.
(b) Verify that, in general, $0.817 \leq \tau_{oct} / \tau_{max} \leq 0.942$. (12+8)

4. (a) Derive the compatibility equations for plane stress conditions.
(b) Develop the bi-harmonic equation in cartesian coordinates. (12+8)

5. For an inclined line load on surface, prove that the radial stress in soil over the circumference of a circle drawn with any diameter along load axis and passing through the load point, is constant. (20)

6.(a) Show that the system of equations defining stability of soil structures is statically determinate.
(b) Prove that in plane problems at the limiting equilibrium,

$$\sigma_x = \sigma (1 + \sin \phi \cos 2\theta) - \psi$$
$$\sigma_z = \sigma (1 - \sin \phi \cos 2\theta) - \psi$$
$$\tau_{xz} = \sigma \sin \phi \sin 2\theta$$

in which, $\sigma = (\sigma_x + \sigma_z)/2 + \psi$, $\psi = c \cot \phi$, ϕ = angle of internal friction, and, θ = angle between the major principal stress direction and the x-axis. (5+15)

7.(a) What are the different kinds of boundary value problems encountered in geotechnical engineering that can be solved by the method of characteristics? Discuss the solution procedure of these problems.
(b) Explain what you understand by 'slip lines'? Give the differential form of the slip lines at a point. (15+5)

8.(a) Give the final equations due to Sokolovsky. Also derive the Sokolovsky's recurrence relations for x ,z, σ and θ.
(b) Why are ξ - η characteristics called 'slip lines'? (15+5)