BENGAL ENGINEERING AND SCIENCE UNIVERSITY, SHIBPUR

M.E. 1st SEMESTER (EM) FINAL EXAMINATION, 2013

THEORY OF VIBRATIONS – I (AM-927)

Full Marks: 70 **Time: 3 hrs**

Answer any FIVE questions.

1. a) Determine the natural frequency of oscillation of the system shown in Fig. Q1a. J is the moment of inertia of the integral pulley about its axis of rotation.
   b) Derive an expression for the natural frequency of the system shown in fig. Q1b. The mass of each pulley is small compared with mass m and therefore, can be ignored. Furthermore, the cord holding the mass is inextensible and has negligible mass. [7+7]
2. a)The system shown in Fig. Q2a has two rigid uniform beams of lengths l and mass per unit length m hinged at the middle and resting on rollers at the test stand. The hinge is restrained from rotation by a torsional spring K_t and supports a mass M held up by another spring k to a position where the bars are horizontal. Determine the equation of motion.
   b) Show that the normal modes of a system are orthogonal with respect to the mass matrix. [8+6]
3. Derive the response for a sinusoidal pulse applied to an undamped spring-mass system as shown in Fig. Q3. [14]
4. A torsional system shown in Fig. Q4 is composed of a shaft of stiffness K_1, a hub of radius r and moment of inertia J_1, four leaf springs of stiffness k_2, and an outer wheel of radius R and moment of inertia J_2. Set up the differential equations for torsional oscillation, assuming one end of the shaft to be fixed. Derive the frequency equation in terms of J_1, J_2, ω_{11} and ω_{22} which are uncoupled frequencies given by the expressions $\omega_{11} = K_1/J_1$ and $\omega_{22} = (4k_2R^2)/J_2$. [14]
5. a) State the reciprocity theorem for the flexibility influence coefficients and prove it.
   b) An airfoil section to be tested in a wind tunnel is supported by a linear spring k_L and a torsional spring k_T as shown in Fig. Q5b. If the center of gravity of the section is a distance e ahead of the point of support, determine the differential equations of motion of the system. [7+7]

6. Assuming a static deflection curve

$$y(x) = y_{max}\left[3\left(\frac{x}{l}\right) - 4\left(\frac{x}{l}\right)^3\right], \quad 0 \le x \le \frac{l}{2}$$

determine the lowest natural frequency of a simply supported beam of constant EI and a mass distribution of $m(x) = m_0 \frac{x}{l}\left(1 - \frac{x}{l}\right)$ by the Raleigh method. [14]

7. Derive the differential equation of motion for the longitudinal vibration of a uniform bar with modulus of elasticity E, material density ρ and length l. Find out the frequency equation for a free-free rod. [8+6]

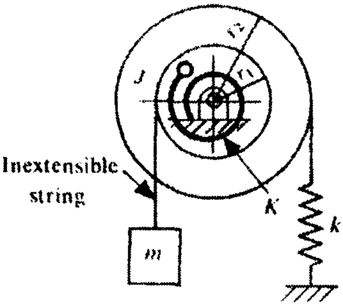

Fig. Q1a

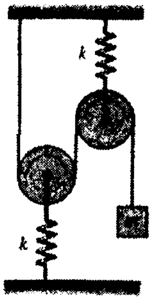

Fig. Q1b

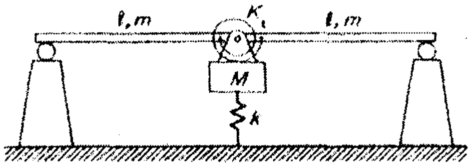

Fig. Q2a

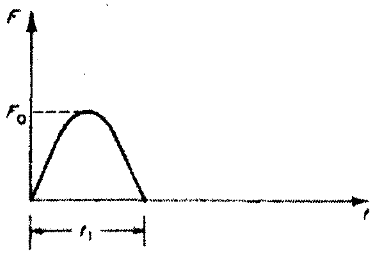

Fig. Q3

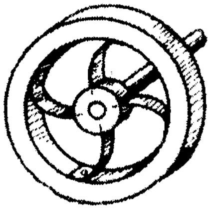

Fig. Q4

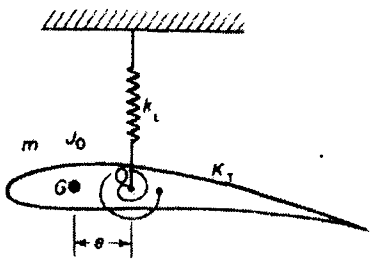

Fig. Q5b