M.E. (EE) 1st semester Final, Examination , November, 2011
Subject : State Variable Analysis
EE 901

Full Marks: 70 **Time: 3 Hrs**

i) Answer five questions taking at least one from Group A
ii) Questions are of equal value
iii) Credit will be given to brief and to the point answers

GROUP A

1. (a) Find the equation of the phase trajectory of the system

$\ddot{\theta} + \sin\theta = 0$ for initial condition $\theta_{t=0} = \dfrac{\pi}{6}$, $\dot{\theta}_{t=0} = 1\sec^{-1}$

(b) For a perturbed second order system $\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$

If $a = 0$ and $b > 0$ find out the nature of the phase plane near the equilibrium point.

(c) Find the linear perturbed motion of the system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_1^3 - x_1 - x_2 - x_3 \end{bmatrix}$

in the neighborhood of the equilibrium point $\overline{X}_e = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

[3+5+6]

2. (a) Define the state variable of a system.

(b) What are isoclines? For the system $\ddot{x} + \dot{x} + x = 0$, find the equation of the isocline where $\dfrac{dx_2}{dx_1} = 1$

(c) For the system $\ddot{y} + \dot{y} + y - y^2 = 0$, classify the equilibrium point $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

[2+4+8]

3. (a) Given the system equation as $\ddot{y} - \varepsilon(1-y^2)\dot{y} + y = 0$, $\varepsilon = 0.1$, can you establish that there is a limit cycle existing in the phase plane of the motion?
(b) For the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -2\sin x_1 - 0.2x_2$$

Show that equilibrium point $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is asymptotically stable by selecting a suitable Lyapunov function and find out the region of stability.

(c) Describe briefly the variable gradient method to obtain a suitable Lyapunov function.

[4+4+6]

GROUP B

4. (a) Find the state transition matrix of:

$$\dot{X}(t) = \begin{bmatrix} t & 0 \\ 0 & 1 \end{bmatrix} X(t);$$

(b) Comment on the stability of the above system.

(c) Prove that if $\phi(t,t_0)$ is the state transition matrix of 4. (a) then

$$\phi(t_0,t) = \phi^{-1}(t,t_0)$$

[6+4+4]

5. (a) Obtain a state space representation of the system given by:

$$G(s) = \frac{(s+1)}{(s^2+5s+6)}$$

Also find the state response, X(t), to a unit step input applied at t=0, taking $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Identify the modes of the system.

(b). What is an "irreducible" realization? Explain in two sentences.

[4+4+2+4]

6. a) Find a set of linearly independent eigen-vectors for the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

b) Suggest how to construct an (n-1)th order observer for a state space realization. How and under what condition does the estimation error go to zero?

[6+5+3]

7.a) Justify that LSVF control allows arbitrary pole placement. What condition should the system fulfill?

b) Design a full order observer for the system below so that the estimation error decays faster than e^{-10t}:

$$\dot{X}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U(t)$$
$$y(t) = \begin{bmatrix} 2 & -1 \end{bmatrix} X(t)$$

[5+2+7]

8. a) Suppose the system in 7. (b) is preceded by a sampler (with T = 1 sec) and a zero order hold. Find the state transition matrix of the discrete time state space model from U(kT) to y(kT).

Is the discrete time system stable?

b) Below are given the state equations of a multivariable system. Check whether it is controllable or not:

$$\dot{X}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} X(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} U(t)$$

c) State the Linear Quadratic Regulator problem in Optimal Control and its solution.

[5+5+4]