ME-ICE, Part I, 1st Semester Final Semester Examination, December 13
Paper Code: Information Theory and Coding (ICE 902)

Full Marks: 70 | Time: 3 Hrs.

(Answer any **Five** questions)

1. (a) A discrete memoryless source has three inputs and two outputs. Determine the mutual information and entropy of the source.
(b) Design channel capacity model for a noisy channel.
(c) Calculate entropy for the source emitting following symbols with the probabilities tabulated below.

xi	Pi
A	½
B	¼
C	1/8
D	1/20
E	1/20
F	1/40

6 + 4 + 4 = 14

2. (a) An independent-symbol binary source with probabilities 0.25 and 0.75 is transmitted over a BSC with transition (error) probability p = 0.01. Calculate the equivocation H(X/Y) and the average mutual information I(X, Y).
(b) Let X be a random variable that adopts values in the range $A = \{x_1, x_2, \ldots, x_M\}$ and represents the output of a given source. Show that, $0 \leq H(X) \leq \log_2(M)$.
(c) An analogue channel perturbed by AWGN has a bandwidth B = 25 kHz and a power signal-to-noise ratio SNR of 18 dB. What is the capacity of this channel in bits per second?

6 + 4 + 4 = 14

3. (a) Perform Huffman encoding on the following eight messages with M = 8 having probabilities given as 0.46, 0.15, 0.10, 0.10, 0.07, 0.05, 0.03, 0.02. Calculate average code length and code efficiency.
(b) Find if the above encoding is fully decodable.
(c) What are the design criteria for an ideal binary source encoder.
(d) What type of source encoding is preferable, Comma code or Prefix free tree code?

6 + 2 + 4 + 2 = 14

4. Let $\mathbf{g(x)} = x^4 + x^3 + x + 1$ and $\mathbf{d(x)} = x^8 + x^7 + x^5 + x^4 + x^3 + x + 1$.
(a) Perform polynomial long division of d(x) by g(x), computing the quotient and remainder.
(b) Draw the circuit configuration for dividing by g(x).

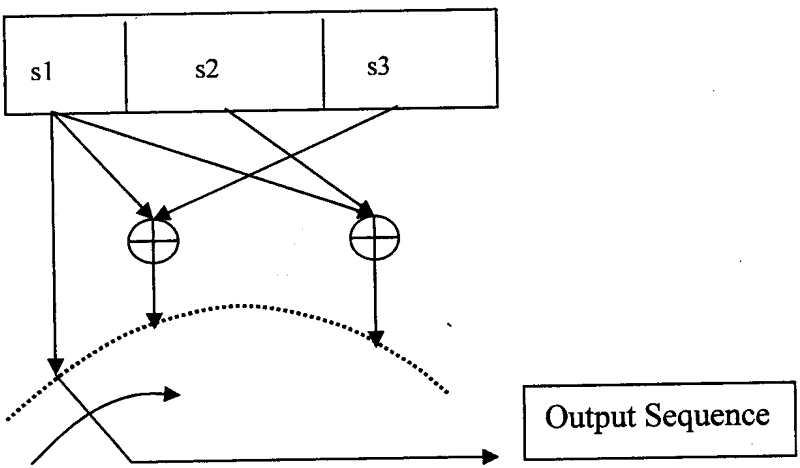
(c) Trace the operation of the circuit for the **g(x)** and **d(x)** given, identifying the polynomials represented by the shift register contents at each step of the algorithm. Also, identify the quotient and the remainder produced by the circuit.
(d) Two polynomials of order 3 and 4 are multiplied and the multiplication result is stored in a register. What is the length of the register for storing the results without any loss of data?

$4 + 2 + 4 + 4 = 14$

5. (a) A cyclic code has size (7, 3). How many unique codeword can be generated from the present code?
(b) The generator polynomial is $g(x) = x^3 + x^2 + 1$ for the (7, 4) systematic code. Find the parity check polynomial.
(c) Design the codeword for the above (7, 4) hamming code.

$2 + 4 + 8 = 14$

6. (a) Draw the output sequence for the data input 11010100, using following encoder.



(b) Draw the tree diagram for the above convolutional encoder. Determine its code rate.
(c) State differences between convolutional coding and block coding.

$4 + 8 + 2 = 14$

7. (a) Define minimal polynomial in Galois field and narrow sense BCH code.
(b) Construct the field GF(8) using the primitive polynomial $p(x) = 1 + x^2 + x^3$, producing a table of vector representation and power representation. Use β to represent the root of $p(x) : \beta^3 + \beta^2 + 1 = 0$.
(c) For a BCH code, let $n = 31$, and $t = 3$ error correcting BCH code in GF(2), with $b = 1$ and β as the primitive element. What will be the possible roots of the polynomial? Find also the conjugacy classes in β.

4 + 6 + 4 = 14

8. For the parity check matrix given below,

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(a) Construct tanner graph
(b) Determine minimum girth of the girth cycle.
(c) Determine the generator matrix
(d) Draw the block diagram of a Turbo encoder and explain its functionality.

2 + 2 + 4 + 6 = 14