INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

B.E. 4th SEMESTER FINAL EXAMINATIONS, 2014
Mathematics- IV-2
(MA-401/2)

Full Marks: 70 **Time: 3 hrs**

Branch : Aerospace Engineering

Use separate answer script for each half

First half

Answer any Four questions, taking at least one from question no. 5 and 6 of this half

1. a) Site an example in each of the following cases :

Fredholm Integral equation of the first kind, Volterra integral equation of the second kind, Singular integral equation, Separable kernel, Transpose equation of a Fredholm integral equation of the second kind.

b) Prove that eigen functions $f(x)$ and $g(x)$ corresponding to distinct eigen values λ_1 and λ_2 respectively of the homogeneous integral equation $f(x)=\lambda\int_a^b K(x,t)\, f(t)\, dt$ and its transpose are orthogonal. **(5+5)**

2. a) Explain how to solve the Fredholm integral equation
$u(x) = f(x) + \alpha \int_a^b K(x,t)u(t)\, dt$ by iterated scheme. What is the condition of convergence?
b) Solve the following Fredholm Integral equation of the second kind :
$$f(x) = x + \int_0^1 (x+y)f(y)dy$$ **(6+4)**

3. a) Determine the solution of the integral equation $f(x)=\int_0^x \frac{g(t)}{(x-t)^\alpha}\, dt,\ 0<\alpha<1$, where $g(t)$ is the only unknown quantity.

b) Solve the Fredholm integral equation $f(x) = 1 + \lambda \int_0^1 (1-3xt)\, f(t)\, dt$ by iterative method. **(5+5)**

4. a) Convert the initial value problem $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 5\sin x,\ \ y(0) = 1,\ y'(0) = -2$ into a Volterra integral equation.

b) If $y(x) = \int_0^1 K(x,t) f(t)\, dt$,

where $K(x,t) = t(x-1)$ *when* $t < x$ *and* $x(t-1)$ *when* $t > x$, show that $y(x)$ satisfies the boundary value problem $y''(x) = f(x),\ y(0) = y(1) = 0.$

5. a) For the functional $I(y) = \int_a^b F(x, y, y')dx$, write down Euler's equation for the path on which the functional have an extreme value . If F is a function of y *and* y' only, show that Euler's equation can be written as $F - y' F_{y'} = C$, where C is a constant.

b) Find the curve with fixed boundary points such that its rotation about the axis of abscissa (i.e. x-axis) give rise to a surface of revolution with minimum surface area. **(3+7)**

6. a) For the extremum of the functional

$$I[y(x)] = \int_a^b F(x, y(x), y'(x), y''(x), \ldots\ldots\ldots\ldots, y^n(x))\, dx$$ with the given boundary

conditions $y(a) = y_0,\ y^k(a) = y_k,\ k = 1, 2, \ldots\ldots, n-1$

$y(b) = z_0,\ y^k(b) = z_k,\ k = 1, 2, \ldots\ldots, n-1$

where the superscripts represent derivative with respect to arguments, derive Euler-Poisson equation.

b) Find the extremal of the functional $\int_0^1 (y'^2 + 12xy)dx,\ y(0) = 0,\ y(1) = 1$. What is the value of the functional on the extremal ?

c) On which curve the functional $\int_0^{\pi/2} (y'^2 - y^2 + 2xy)dx,\ y(0) = 0,\ y(\pi/2) = 0$ be extremize ? **(4+3+3)**

Second Half
(Answer any three questions)

7. (a) A function $f(z) = u(x,y) + iv(x,y)$ is analytic in a region G iff v is the harmonic conjugate of u in G.
(b) Show that $f(z) = |z|^2$ is continuous everywhere but it is nowhere differentiable except at the origin.
(c) Show that the function:

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, z \neq 0$$
$$= 0, \qquad z = 0$$

is not differentiable at the origin even though it satisfies C-R equations there.

Or,

(d) Let $u(x,y) = e^x cosy$. Determine a function $v(x,y)$ such that $f(z) = u(x,y) + iv(x,y)$ is analytic.

8. (a) State and prove Cauchy's integral formula.
(b) Evaluate the following integral using Cauchy's integral formula:

$$\int_c \frac{4-3z}{z(z-1)(z-2)} dz,$$

where $'c'$ is the circle $|z| = \frac{3}{2}$.
(c) Evaluate the integral, $I = \int_\gamma z^2 dz$,
where γ is along the x-axis from 0 to 1 and then along the line parallel to y-axis from 1to $1 + 2i$.

9. (a) If l be the length of a rectifiable curve γ and M is a positive number such that $|f(z)| \leq M \ \forall z \in \gamma$, then prove that: $\left|\int_\gamma f(z)dz\right| \leq Ml$.
(b) Without evaluating the integral show that $\left|\int_C \frac{dz}{z^4}\right| \leq 4\sqrt{2}$ where C is the line segment from $z = i$ to $z = 1$.
(c) Define "Isolated singular point" of a function f. Find all the singularities of:
(i) $cosec\frac{\pi}{z}$, (ii) $\frac{sinz}{z}$.

10. (a) If f is analytic on the disk $D:|z-\alpha| < R$, then prove that at each point $z \in D$, f has the series representation:

$$f(z) = \sum_{n=0}^{\infty} a_n(z-\alpha)^n, \text{ where}$$

$$a_n = \frac{f^n(\alpha)}{n!}, n = 0,1,2,\ldots$$

(b) Evaluate by the method of contour integration (any one):
(i) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$, (ii) $\int_0^{\infty} \frac{dx}{1+x^4}$.