B.Arch First Semester
Subject: Mathematics-IA (MA-101A)

Full marks-70 **Time-3 hrs**

1st half

1 mark is reserved for general proficiency.

Answer Q.No.-1 and any three questions from the rest.

1. Answer any two questions:
   (a) Sate Cauchy-Hadamard theorem for power series.
   (b) State Raabe's test.
   (c) State Cauchy's mean value theorem.
   (d) Write the expression of radius of curvature in polar co-ordinates.

[2 × 2 = 4]

2. (a) State and prove Leibnitz's theorem on the nth derivative of the product of two functions of x.

   (b) If $f(x)=x^n$, prove that

$$f(1)+\frac{f'(1)}{1!}+\frac{f''(1)}{2!}+\frac{f'''(1)}{3!}+\ldots+\frac{f^n(1)}{n!}=2^n .$$

[5 + 5 = 10]

3. (a) If $f(h) = f(0) + h\,f'(0) + \frac{h^2}{2!}\,f''(\theta h),\ 0 < \theta < 1,\ f(x) = \frac{1}{1+x}$ *and* $h = 7$, find θ.

   (b) Expand the function e^x in power of x as an infinite series.

[5 + 5 = 10]

4. (a) Discuss its convergence for different values of p in p-series.
   (b) Test the convergence of the series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+\cdots\cdots$

[5 + 5 = 10]

5. (a) Determine the radius of convergence of the power series $\sum\frac{(n+1)x^n}{(n+2)(n+3)}$.

(b) Find the radius of curvature of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta, b\sin\theta)$.

[5 + 5 = 10]

6. (a) State Rolle's theorem. Give the geometric interpretation of Rolle's theorem. Show that Rolle's theorem is not applicable to $f(x)=\tan x$ in $[0, \pi]$, although $f(0)=f(\pi)$.

(b) State Lagrange's mean value theorem. Use mean value theorem to prove the inequality: $0<\frac{1}{x}\log_e\frac{e^x-1}{x}<1$ for x>0.

[5 + 5 = 10]

2nd half

One mark is reserved for general proficiency

Answer Q.No.-7 and any three questions from the rest.

7. Answer any two questions:
   (a) Define relative error.
   (b) State Taylor's theorem for two variables.
   (c) Find the minimum value of $x+y$ subject to the condition $\frac{1}{x}+\frac{1}{y}=\frac{1}{5}$, $x,y>0$.
   (d) Evaluate $\iint_D (4xy-y^2)\,dx\,dy$ where D is the rectangle bounded by $x=1, x=2$, $y=0, y=3$.

[2 × 2 = 4]

8. (a) Define homogeneous function of two variables. State and prove Euler's theorem on homogeneous function of two variables.

(b) If $u=\sin^{-1}\left[\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}\right]$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{20}\tan u$.

[5 + 5 = 10]

9. (a) If $x=e^{r\cos\theta}.\cos(r\sin\theta), y=e^{r\cos\theta}.\sin(r\sin\theta)$, prove that $\frac{\partial x}{\partial r}=\frac{1}{r}\frac{\partial y}{\partial\theta}, \frac{\partial y}{\partial r}=-\frac{1}{r}\frac{\partial x}{\partial\theta}$. Hence prove that $\frac{\partial^2x}{\partial r^2}+\frac{1}{r}\frac{\partial x}{\partial r}+\frac{1}{r^2}\frac{\partial^2x}{\partial\theta^2}=0$.

(b) The resonant frequency in a series electrical circuit is given by $f = \frac{1}{2\pi\sqrt{LC}}$. If the measurements of L and C are in error by 2% and -1 % respectively, find percentage error in f.

[5 + 5 = 10]

10. (a) Given $2^x.3^y.5^z = 1$ find the maximum value of $(x+1)(y+1)(z+1)$.

(b) Using Lagrange's multiplier method show that the largest rectangle with a given perimeter is a square.

[5 + 5 = 10]

11. (a) Expand $\frac{1}{xy}$ about $(1,1)$ by Taylor's series expansion up to second degree terms.

(b) Find the volume of the tetrahedron in space cut from the first octant by the plane $6x + 3y + 2z = 6$.

[5 + 5 = 10]

12. (a) Find the mass, co-ordinates of the centre of gravity (centroid) and moments of inertia relative to x-axis, y-axis and origin of rectangle $0 \le x \le 5, 0 \le y \le 3$ having mass density x^2y^2.

(b) Evaluate $\iint_D (x^2 + y^2)dx\, dy$ where D is the region in the first quadrant bounded by

$x^2 - y^2 = a, x^2 - y^2 = b, 2xy = c, 2xy = d, 0 < a < b, 0 < c < d$.

[5 + 5 = 10]