Bengal Engineering and Science University, Shibpur
Department of Aerospace Engineering and Applied Mechanics
B.E. (AE) Part-III 6th Semester Final Examination, April-May 2013
ORBITAL MECHANICS (AE-605)

Time: 3 hours | Full Marks: 70

Answer any FOUR questions.
Assume reasonable data, if not supplied with the problem.
The questions are of equal marks.

1. a) Derive the expressions for mean distance $\bar{r}_\theta$ and time-averaged radius $\bar{r}_t$ of an elliptical orbit in terms of semi-major axis a and eccentricity e and hence, express the relationship between $\bar{r}_\theta$ and $\bar{r}_t$.
   b) Calculate the radius r at which the speed on a hyperbolic trajectory is 1.1 times the hyperbolic excess speed in term of the periapsis radius r_p and eccentricity e.

2. a) A spacecraft on a parabolic trajectory around the earth has a perigee radius of 7500 km. i) How long does it take to coast from $\theta = -90\ degrees$ to $\theta = +90\ degrees$? ii) How far is the spacecraft from the center of the earth 24 hours after passing through perigee?
   b) Show that the direction cosine matrix (DCM) is an orthogonal matrix.

3. a)Derive Kepler's equation in terms of mean anomaly (M_e), eccentric anomaly (E) and eccentricity (e) for an elliptical orbit.
   b) A satellite is in earth orbit for which the perigee altitude is 200 km and the apogee altitude is 600 km. Find the time interval during which the satellite remains above an altitude of 400 km.

4. a) Derive an expression for the orbital specific energy in terms of the gravitational parameter μ, and the orbital constants h and e.
   b) Find the minimum additional speed required to escape from the geostationary equatorial orbit (GEO).
   c) Calculate the velocity, relative to the earth, required to escape the solar system on a parabolic path from the earth's orbit. Mass of the sun is 1.989×10^{30} kg and the distance of the earth's center from the center of the sun is 149.6×10^6 km.

5. For a spacecraft, the following orbital parameters are given: $e = 1.2$; perigee altitude = 200 km; $i = 50^o$; $\Omega = 75^o$; $\omega = 80^o$. Calculate $\bar{r}$ and $\bar{v}$ at perigee relative to a) the perifocal reference frame and b) the geocentric equatorial reference frame.

6. At a given instant a space object has the following position and velocity vectors relative to an earth-centered non-rotating frame of reference: $\bar{r_o} = 3450\,\bar{\imath} - 1700\,\bar{\jmath} + 7750\,\bar{k}\ (km)$ and $\bar{v_o} = 5.4\,\bar{\imath} - 5.4\,\bar{\jmath} - 1.0\,\bar{k}\ (km/s)$. Find the position vector $\bar{r}$ and velocity vector $\bar{v}$ after the true anomaly changes by 75 degrees.

7. At a time t_0 the position $\bar{r}$ and velocity $\bar{v}$ of a satellite in the geocentric equatorial frame are: $\bar{r} = 6472.7\,\bar{I} - 7470.8\,\bar{J} - 2469.8\,\bar{K}\ (km)$ and : $\bar{v} = 3.9914\bar{I} + 2.7916\,\bar{J} - 3.2946\,\bar{K}\ (km/s)$. Find the orbital elements.