

**Dynamics for Aerospace Engineers (AE 404)**

Time: 3 Hrs.

Full Marks: 70

All questions are of equal value

**Separate Answer scripts are not required**

**Group A**

Answer any FOUR of the following:

1. The circular disk B of radius  $r$  rolls without slipping in a circle of radius  $b$  on the disk C that rotates with a constant angular velocity  $\Omega$  about the  $z$ -axis when viewed from above. The arm OA rotates as shown in the figure Q. 1. Determine the expression for angular velocity  $\omega$  and angular acceleration  $\alpha$  of the disk B. 14
  
2. For the instant represented collar B is moving along the fixed shaft in the  $X$ -direction with a constant velocity  $v_B = 4$  m/s. Also at this instant  $X = 0.3$  m and  $Y = 0.2$  m. Calculate the velocity of collar A, which moves along the fixed shaft parallel to the  $Y$ -axis. Each clevis is free to rotate about the axis of the rod. (Fig. Q.2) 14
  
3. a) Derive the expression for kinetic energy for a rigid body moving with mass centre velocity  $v$  and angular velocity  $\omega$ . 5  
b) The aircraft landing gear viewed from the front is being retracted immediately after takeoff, and the wheel is spinning at the rate corresponding to the takeoff speed of 200 km/h. The 45-kg wheel has a radius of gyration about its  $z$ -axis of 370 mm. Neglect the thickness of the wheel and calculate the angular momentum of the wheel about G and about A for the position where  $\theta$  is increasing at the rate of  $30^\circ$  per second. (Fig. Q. 3 (b)) 9
  
4. a) Derive the expression for the moment of inertia about any axis (OM) in terms of the direction cosines of the axis and the moments and products of inertia about the  $x$ - $y$ - $z$  coordinate axis. 7  
b) The large satellite-tracking antenna has a moment of inertia  $I$  about its  $z$ -axis of symmetry and a moment of inertia  $I_0$  about each of the  $x$ - and  $y$ - axes. Determine the angular acceleration  $\alpha$  of the antenna about the vertical  $Z$ -axis caused by a torque  $M$  applied about  $Z$  by the drive mechanism for a given orientation  $\theta$ . (Fig. Q. 4 (b)) 7
  
5. a) The irregular rod has a mass  $\rho$  per unit length and rotates about the shaft  $z$ -axis at the constant angular velocity  $\omega$ . Determine the bending moment in the rod at A. Neglect the small moment due to the weight of the rod. (Fig. Q.5). 6

- b) If the rod of previous problem (question 5 (a)) starts from rest under the action of a torque  $M_0$  applied to it by the collar at A about the z-axis, determine the initial bending moment  $M$  in the rod at A. Neglect the small moment due to the weight of the rod. (Fig. Q.5). 8
6. a) Derive the expressions for the general equations of rotation of a symmetrical body about either the mass centre or about a fixed point. 7
- b) Derive the approximate expression for precession of steady-state gyroscopic motion when spin motion is much larger compared to the precession motion. 7

## Group B

### Answer any one of the following:

7. a) Prove that  $\sum \lambda_i a_{ij}$  is nothing but the generalized constraint force corresponding to the generalized coordinate  $q_j$  where  $\lambda_i$  s are the Lagrange multipliers and  $\sum a_{ij} \delta q_j = 0$ ;  $i = 1, 2, \dots, (m-n)$  represent the constraint equations. (You need not start from the very beginning and can refer to the relevant mathematical parts only.)  $m$  and  $n$  are the number of generalized coordinates and the degrees of freedom, respectively. 5
- b) The fig. Q. 7 (b) shows a spring mass system that can freely move along the horizontal direction.  $x_1$ ,  $x_2$  and  $x_3$  represent displacements of  $m_1$ ,  $m_2$  and  $m_3$  from their respective equilibrium positions. Derive (i) the expression for the Lagrangian and (ii) the Lagrange's equation of motion. 9
8. a) Mathematically show that workless constraints imply zero generalized constraint forces. 5
- b) Find out the friction force acting on the uniform circular cylinder, of mass  $m$  and radius  $r$ , that is rolling (without slipping) down the inclined surface of the wedge of mass  $M$  that can itself move freely in the horizontal direction on a smooth floor. The inclination of the wedge surface is  $\beta$  and its mass is  $M$ . Use Lagrange multiplier technique. fig. Q: 8 (b) 9

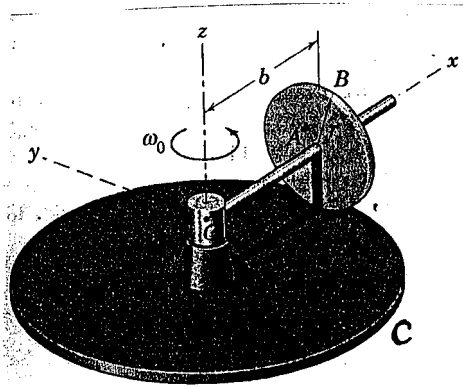


Fig Q. 1

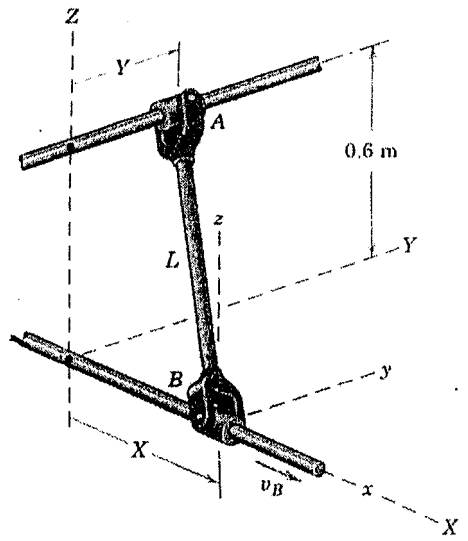


Fig Q. 2

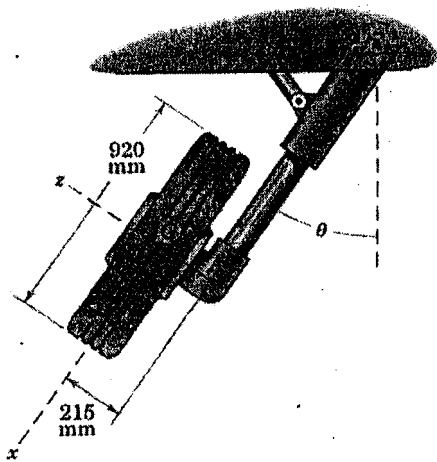


Fig Q. 3 (b)

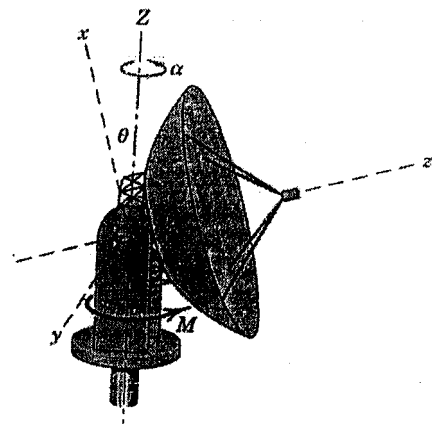


Fig Q. 4 (b)

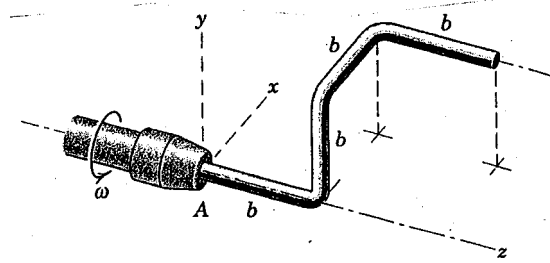


Fig Q. 5

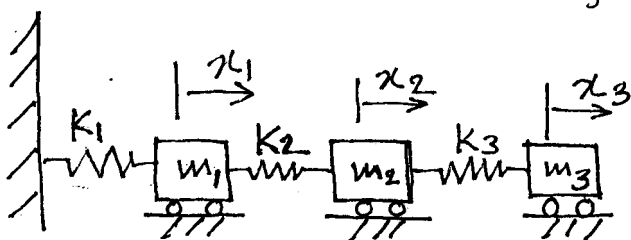


Fig Q. 7 (b)

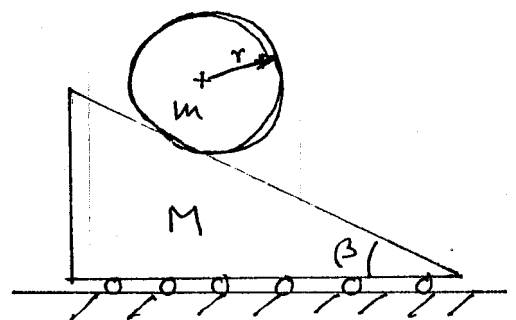


Fig. Q. 8 (b)