B.Arch. Part-I 2nd Semester Examination, 2007

Mathematics - IIA

(MA-201A)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.

FIRST HALF

(*Answer Q.No.1 and TWO from the rest.*
One mark is reserved for general proficiency.)

1. Answer any two questions :

 a) If the perpendicular straight lines $ax + by + c_1 = 0$ and $bx - ay + c_2 = 0$ be taken as the axes of x and y respectively, show that the equation $(ax + by + c_1)^2 - 2(bx - ay + c_2)^2 = 1$ will be transformed into $y'^2 - 2x'^2 = \frac{1}{a^2 + b^2}$.

 b) Reduce the following equation to canonical form and determine the nature of the conic represented by it

$$4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$$

 c) Show that the equation $(a^2+1)x^2 + 2(a+b)xy + (b^2+1)y^2 = c(c>0)$ represents an ellipse of area $\pi c/(ab-1)$. **[6+6+6]**

2. a) Show that the straight lines, whose direction cosines l, m, n are given by $a^2 l + b^2 m + c^2 n = 0$, $mn + nl + lm = 0$, are parallel if $a + b + c = 0$.

 b) A variable plane passes through the point (f, g, h) and meets the axes in A, B, C. If the planes through A, B, C and parallel to the axes meet at ρ, show that the locus of ρ is $f/x + g/y + h/z = 1$. **[5+6]**

3. a) Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$.

 b) A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the locus of the centroid of the triangle ABC is the sphere $q(x^2 + y^2 + z^2) = 4k^2$. **[6+5]**

4. a) If $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ be unit vector satisfying the condition $\bar{\alpha} + \bar{\beta} + \bar{\gamma} = \bar{0}$, then show that $\bar{\alpha}.\bar{\beta} + \bar{\beta}.\bar{\gamma} + \bar{\gamma}.\bar{\alpha} = -3/2$.

b) Find a vector $\bar{a}$ collinear with the vector $\bar{b} = (2, 1, 3)$ satisfying $\bar{a}.\bar{b} = 16$.

c) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$. **[4+3+4]**

5. a) Find a vector of magnitude 11 units and perpendicular to the plane of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$.

b) If α, β are acute angles, using vector method prove that

$$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta.$$

c) A force of 10 units acts in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ and passes through a point $2\hat{i} + 2\hat{j} + \hat{k}$. Find the moment of the force about the point $\hat{i} + \hat{j} + \hat{k}$. **[3+4+4]**

SECOND HALF

(*Answer any THREE questions.*
Two marks are reserved for general proficiency.)

6. a) State the Fundamental theorem of linear programming problem (LPP).

Solve the following LPP graphically :-

$$\begin{aligned} &\text{Minimize } Z = 3x + 5y \\ &\text{subject to } 2x + 3y \geq 12 \\ &\qquad -x + y \leq 3 \\ &\qquad 0 \leq x \leq 4 \\ &\qquad y \geq 3 \end{aligned}$$

b) Solve the following LPP by Simplex method

$$\begin{aligned} &\text{Maximize } Z = 60x + 50y \\ &\text{subject to } x + 2y \leq 40 \\ &\qquad 3x + 2y \leq 60 \\ &\text{and } \quad x,\ y \geq 0 \end{aligned}$$

[6+5]

7. a) The mean and standard deviation of marks of 70 students were found to be 65 and 5.2 respectively. Later it was detected that the mark of one student was wrongly recorded as 85 instead of 58. Obtain the correct standard deviation.

b) For the following grouped frequency distribution find the median and mode

Class-interval :	130-134	135-139	140-144	145-149	150-154	155-159	160-164
Frequency :	5	15	28	24	17	10	1

[6+5]

8. a) Calculate Karl Pearson's co-efficient of skewness from the following data :

Variable :	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
Frequency :	12	18	35	42	50	45	30	8

b) Fit (i) a linear and (ii) a quadratic curve to the following data by Least Square Polynomial Approximation.

x	4	6	8	10	12
y	13.72	12.90	12.01	11.14	10.31

[5+6]

9. a) Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

b) Solve : $\frac{d^2y}{dx^2} + y = 0$

when $x = 0$, $y = 4$ and when $x = \pi/2$, $y = 0$

c) Solve : $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = x^4$. [4+3+4]

10. a) If $\frac{d^2x}{dt^2} + 4x = 0$ and $x = 3$, $\frac{dx}{dt} = 8$ at $t = 0$

find x in terms of t.

b) Solve : $\frac{d^2y}{dx^2} - y = xe^{3x}$

c) Solve : $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$. [3+4+4]

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