B.Arch. Part-I 2nd Semester Examination, 2009

Mathematics-IIA
(MA-201A)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.
Answer SIX questions, taking THREE from each half.
Two marks are reserved for general proficiency in each half.

FIRST HALF

1. a) If by the orthogonal transformation without change of origin the expression $ax^2+by^2+2hxy+2gx+2fy+c$ be changed into $a'x^2+b'y^2+2h'xy+2g'x+2f'y+c'$, then prove that $a'b'-h'^2=ab-h^2$.

   b) Find the angle through which the axes are to be rotated so that the equation $\sqrt{3}x+y+6=0$ may be reduced to the form $x=c$. Also determine the value of c. [7+4]

2. a) Reduce the following equation to its canonical form and determine the nature of the conic represented by it.
   $$x^2-6xy+y^2-4x-4y+12=0.$$

   b) Show that the straight lines whose direction cosines are given by $al+bm+cn=0$, $fmn+gnl+hlm=0$ are perpendicular if $f/a+g/b+h/c=0$. [7+4]

3. a) A variable plane which is at a constant distance $3p$ from this origin O cuts the axes in A, B, C respectively. Show that the locus of the centroid of the ΔABC is
   $$\bar{x}^2+\bar{y}^2+\bar{z}^2=\bar{p}^2.$$

   b) Find the image of the point (3, –8, 4) after reflection in the plane $6x-3y-2z+1=0$. [5+6]

4. a) Show that the straight lines $x=ay+b, z=cy+d$ and $x=a'y+b', z=c'y+d'$ are at right angles if $aa'+cc'+1=0$.

   b) A sphere of constant radius r passes through the origin and cuts the axes in A, B, C respectively. Prove that the locus of the foot of the perpendicular from 0 to the plane ABC is given by $(x^2+y^2+z^2)(\bar{x}^2+\bar{y}^2+\bar{z}^2)=4r^2$. [4+7]

5. a) If $\bar{a}=\hat{i}\cos\alpha+\hat{j}\sin\alpha$, $\bar{b}=\hat{i}\cos\beta+\hat{j}\sin\beta$ find $\bar{a}.\bar{b}$ and hence show that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

   b) A rigid body is rotating with a speed of 1.5 radians per second about an axis OR where R is the point $2\hat{i}-2\hat{j}+\hat{k}$ relative to 'O'. Find the velocity of the particle of the body at the point $4\hat{i}+\hat{j}+2\hat{k}$ [5+6]

SECOND HALF

6. a) Solve : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0.$

   b) Solve : $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 54y = 0.$

7. Solve the following differential equations :

   a) $\frac{d^4y}{dx^4} + a^4y = 0.$

   b) $\frac{d^2y}{dx^2} + 4x = 0$ and $x=3,\ \frac{dx}{dt} = 8$ at $t=0.$

8. Solve : a) $(D^2 - 1)y = e^{2x}.$

   b) $(D+1)(D-2)y = e^{-x}.$

9. Solve the following differential equations :

   a) $\frac{d^2y}{dx^2} + 9y = 5x^2.$

   b) $(D^4 - 8D)y = x^2 + e^{3x}.$

10. a) Solve : $(3D^2 + 2D - 8)y = 5\cos x.$

    b) Solve : $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = e^{2x} + e^{x} + 3e^{-x}.$