B.E.(Arch) Part-I 1st Semester Final Examination 2011

Mathematics-IA
(MA-101A)

Time-3 hours   Full Marks:70

Use separate answerscript for each half.
Answer SIX questions, taking THREE from each half.
Two marks are reserved for general proficiency in each half.

FIRST HALF

1. (a) State Euler's theorem for homogeneous functions. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.

b) If $y = \cos(m\sin^{-1} x)$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. Also, find the value of y_n when x=0.

5+6=11

2. (a) State Lagrange's Mean Value Theorem and give its geometrical interpretation. Show that the Lagrange's remainder after n terms in the expansion of $e^{ax}Cos(bx)$ in powers of x is $\dfrac{(a^2+b^2)^{\frac{n}{2}}}{n!}x^n e^{a\theta x} Cos\left(b\theta x + n\tan^{-1}\dfrac{b}{a}\right)$, $0<\theta<1$.

(b) Expand in infinite series in powers of x, stating the condition under which the expansion is valid, the function $f(x) = \log(1+x),\ x > -1$.

6+5=11

3. (a) If $x\cos\alpha + y\sin\alpha = p$ touches the curve $\dfrac{x^m}{a^m} + \dfrac{y^m}{b^m} = 1$, then show that

$$(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$$

(b) Find the equations of the tangents to the conic $x^2 + 4xy + 3y^2 - 5x - 6y + 3 = 0$ which are parallel to the straight line $x + 4y = 0$.

6+5=11

4. (a) Find the radius of curvature at the point (r,θ) on the cardioide $r = a(1-\cos\theta)$ and show that it varies as $\sqrt{r}$.