B.Arch. Part-I 1st Semester Examination, 2009-10

Mathematics-IA

(MA-101A)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.

Answer SIX questions, taking THREE from each half.

Two marks are reserved for general proficiency in each half.

FIRST HALF

1. a) If $y = \sin kx + \cos kx$, prove that $y_n = k^n\{1 + (-1)^n \sin 2kx\}^{½}$.

   b) If $y = \cos(m \sin^{-1} x)$,
   prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$.
   Hence find the value if $(y_n)_{x=0}$. [4+(4+3)]

2. a) State Lagrange's mean value theorem. Give its geometrical interpretation.
   Apply mean value theorem to show that $0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1$.

   b) In the MVT $f(h) = f(o) + hf'(\theta h)$, $0 < \theta < 1$, prove that $\lim_{h\to 0+} \theta = \frac{1}{2}$,
   where $f(x) = \cos x$. [8+3]

3. a) State and prove Taylor's theorem with Cauchy's form of remainder.

   b) Expand $\log(1+x)$ is powers of x with Cauchy's form of remainder. [(2+5)+4]

4. a) Find the value of $\lim_{x\to 0} (\sin x)^{2\tan x}$.

   b) Find the values of p and q such that
   $\lim_{x\to 0} \frac{x(1-p\cos x)+q\sin x}{x^3} = \frac{1}{3}$. [5+6]

5. a) Find the radius of curvature of $x^{2/3} + y^{2/3} = a^{2/3}$ at any point (x, y).

   b) Find the asymptotes of the curve
   $x^2y^2 - a^2(x^2+y^2) - a^3(x+y) + a^4 = 0$. [5+6]

SECOND HALF

6. a) Prove that $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$ if $A+B+C=\pi$.

b) Show that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$ [5+6]

7. a) If $|A|=2$ and adj $A = \begin{bmatrix} -2 & 3 & 1 \\ 6 & -8 & -2 \\ -4 & .7 & 1 \end{bmatrix}$, then find A.

b) Determine the rank of the following matrix by reducing it to its normal form or Echelon form :

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & -2 \\ 3 & 6 & -5 & 10 \end{bmatrix}$$

[5+6]

8. a) Show that the series $\sum \frac{1}{n^p}$ is convergent for $p>1$ and divergent for $p \leq 1$.

b) Examine the nature of convergence of the series :

$$\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \ldots\ldots$$

[6+5]

9. a) Solve the equations by Cramer's Rule :

$$x + 2y + 5z = 23$$
$$3x + y + 4z = 26$$
$$6x + y + 7z = 47.$$

b) Show that a square matrix A can be expressed uniquely as a sum of a symmetric matrix and a skew-symmetric matrix.

c) Define orthogonal matrix. Prove that orthogonal matrices are non-singular.

[4+4+3]

10. Examine the convergence or divergence of the following series :

a) $\sum u_n$ where $u_n = (n^3+1)^{1/3} - n$.

b) $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \ldots\ldots$ [6+5]

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