19.1.9

Ex/BESUS/ MA-101A/09

B.Arch. Part-I 1st Semester Examination, 2009

Mathematics-IA

(MA-101A)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.
Answer SIX questions, taking THREE from each half.
The questions are of equal value.
Two marks are reserved for general proficiency in each half.

FIRST HALF

1. a) State and prove Leibnitz's theorem for the nth derivative of the product of two functions.

   b) If $y=(x+\sqrt{1+x^2})^m$, then prove that
   $(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$.
   Hence find $(y_n)_0$.

2. a) State Lagrange's form of mean value theorem and give its geometrical interpretation.

   b) If $f(h)=f(o)+hf'(o)+\frac{h^2}{2!}f''(\theta h)$, $0<\theta<1$, find θ, when $h=1$ and $f(x)=(1-x)^{5/2}$.

3. a) In the Cauchy's mean value theorem if $f(x)=e^x$ and $g(x)=e^{-x}$, show that θ is independent of both x and h and is equal to ½.

   b) State Taylor's theorem with Lagrange's form of remainder.
   Expand the function $\cos x$ in a finite series in powers of x, with remainder in Lagrange's form.

4. a) Show that if $-1<x<1$, $(1+x)^{1/3}=1+\frac{1}{3}x-\frac{1}{9}x^2+\frac{5}{8!}x^3-\frac{10}{243}x^4+R_5$ where $R_5=\frac{x^5}{5!}\left(1+\frac{\theta}{x}\right)^{-14/3}$, $(0<\theta<1)$.

   b) Show that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots\ldots+\frac{1}{n^p}+\ldots\ldots$
   (i) is convergent is $p>1$
   (ii) is divergent if $p\le 1$.

5. Test the convergence of the following series

(i) $\frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots\ldots$

(ii) $1 + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \frac{(1+\alpha)(2+\alpha)(3+\alpha)}{(1+\beta)(2+\beta)(3+\beta)} + \ldots\ldots$

(iii) $\left[\frac{2^2}{1^2} - \frac{2}{1}\right]^{-1} + \left[\frac{3^3}{2^3} - \frac{3}{2}\right]^{-2} + \left[\frac{4^4}{3^4} - \frac{4}{3}\right]^{-3} + \ldots\ldots$

(iv) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$.

SECOND HALF

6. a) Show that any square matrix can be expressed as sum of symmetric and a skew symmetric matrix.

b) If $3A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$, prove that $A^{-1} = A^T$.

c) If $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ 2 & 3 & 2 \end{pmatrix}$, show that $A^3 - A^2 - A + I = 0$. Hence find A^{-1}.

7. a) Find the inverse of a matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$.

b) Solve the following equations by matrix inversion method.

$x + y - z = 6,\ 2x - 4y + 2z = 1,\ 2x - 3y + z = 1.$

c) Given the system of equations :

$x_1 + 4x_2 + 2x_3 = 1$

$2x_1 + 7x_2 + 5x_3 = 2a$

$4x_1 + 6x_2 + 10x_3 = 2a + 1$

Find for what values of a, b the above system has (i) a unique solution, (ii) no solution, (iii) many solutions.

8. a) Find the rank of the matrix $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

b) Prove without expanding that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2).$$

c) If $ax+by+cz=1,\ cx+ay+bz=0,\ bx+cy+az=0$, find x, y, z and hence prove that the determinants

$$\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} a & c & d \\ b & a & c \\ c & b & a \end{vmatrix}$$

are reciprocal to each other.

9. a) Prove that $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$. (without expanding).

b) Show that $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af-bc+cd)^2$. (without expanding).

c) Using determinant, find for what values of λ, μ the system of equation s $x+y+z=6,\ x+2y+3z=10$ and $x+2y+\lambda z=\mu$ will have (i) unique solution, (ii) so solution, (iii) many solutions.

10. a) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2=4ax$, then show that

$$\rho_1^{-2/3}+\rho_2^{-2/3}=(2a)^{-2/3}.$$

b) Define asymptote of a curve. Show that the asymptotes of the curve $x^2y^2=a^2(x^2+y^2)$ from a square of side 2a.

c) Define envelope of a curve. Find the envelope of the family of ellipses

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

where $a+b=c$, (a, b being two parameters).

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